Only a few techniques matter!

On the number of curves on the wage frontier*

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Summary

Samuelson's surrogate production function seemed discredited after the capital controversy, but empirical investigations have shown that wage curves derived from input-output tables are approximately linear and that reverse capital deepening is rare. This paper provides a novel theoretical explanation of these findings, based on assumptions significantly weaker than that of uniform intensities of capital, and it demonstrates that the number of wage curves of individual techniques, which appear on the envelope, is surprisingly small.

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1. Introduction

Wage curves have become the main tool for the analysis of technical choice, but what does their envelope look like? Joan Robinson used to say that one should expect one technique to dominate all others, independently of distribution. It sounded like a polemical remark to counteract all reference to neoclassical substitution, and she was more cautious in her writing[1], but the drawing on the blackboard looked like fig. 1, w_1, w_2 representing two techniques. (The reader not familiar with the Sraffa analysis and the notation used here can pick it up in the first paragraph of section 2.)

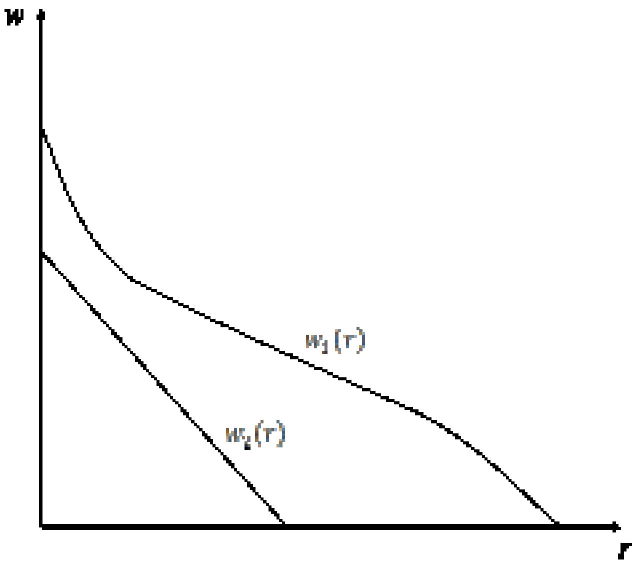

Fig. 1: The choice of technique (Joan Robinson case)

[1] A reflection of her teaching is to be found in Robinson (1979, p. 267), where the better technique implies a rise of both output per head and of the maximum rate of profit. She added: 'This appears to correspond to the typical development most prevalent in modern large-scale industry', and she rejected the idea that the rise of output per head might be bought by a reduction of the maximum rate of profit (ibid., p. 272).

The surrogate production function, on the other hand, looked like this (fig. 2)[2]:

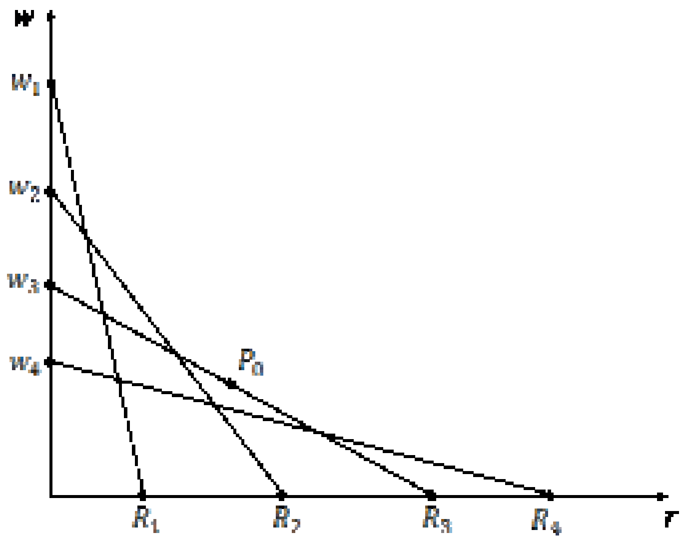

Fig. 2: The choice of technique (Paul Samuelson case)

The maximum rates of profit R_1 of the individual techniques were in reverse order to the wage rates paid at $r = 0$.

Sraffa spoke of a 'rapid succession of switches' of techniques along the envelope, and there was reswitching and reverse capital deepening (fig. 3):

2 The constellation shown in fig. 2 can also be interpreted as a succession of techniques with a rising organic composition of capital, as in Schefold (1976), but Joan Robinson objected; see our subsequent exchange of letters in the Joan Robinson Archive and Robinson (1979, p. 272).

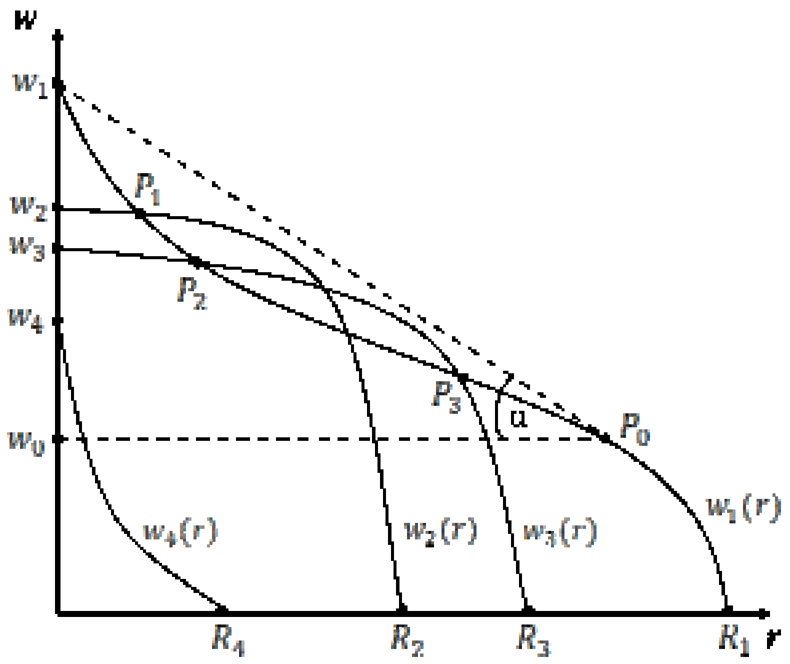

Fig. 3: There is reverse capital deepening at P_3. It would be reswitching, if $w_2(r)$ was not there so that P_2 would be on the envelope. Capital per head at P_0 equals $tg\,\alpha$.

Reverse capital deepening is opposed to the equilibrating mechanism postulated by neoclassical theory. Consider the Samuelson case and suppose the economy is in a state of full employment at P_0 in fig. 2, suppose further that real wages are forced up by trade union action (money wages rise more than prices). It then becomes profitable to use the technique of wage curve $w_2(r)$ for which the intensity of capital is higher. If there is not enough accumulation of capital, unemployment results which reduces trade union power and hence wages, confirming the neoclassical view that there is one level of the real wage at which a full employment equilibrium is stable.

But if the economy is in P_0 in fig. 3, the same rise of the real wage would lead, if profits are maximised, to the adoption of technique $w_3(r)$, to the left of switchpoint P_3, and the intensity of capital would fall. If the amount of capital[3] employed was not reduced, labour demand would be enhanced, encouraging further rises of the wage: the change of factor prices would not stabilise the equilibrium.

The neoclassical equilibrating mechanism is not valid in general already if there is only one example of reverse capital deepening. Could it still be relevant as a rule with exceptions? Not, if the cases with reverse capital deepening are frequent.

The applicability of the critique of neoclassical theory based on reverse capital deepening therefore depends on how often it occurs – if reverse capital deepening occurs very rarely, if it is only a logical possibility and not likely to be encountered in reality, the critique remains academic. How likely is it? Before asking this question, one should ask how many switches there are in the first place.

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3 Fig. 3 shows how the amount of capital to be employed in a steady state at a given rate of profit can be determined. This 'demand' for capital changes in the direction opposed to the hypotheses of neoclassical theory, if there is reverse capital deepening. This paper shows that the objection is less damaging for neoclassical theory (provided it is based on empirical generalisations à la Schmoller and is not meant to hold *a priori* à la Menger) than I, for one, used to think. Another possible objection is that the 'supply of capital' cannot be defined meaningfully in the transition between steady states, as in this thought experiment based on Fig. 3. Objections to the 'supply of capital' also are important; we shall come back to the supply side at the end of section 2.

But the questions are linked. I used to believe in the 'rapid succession' (Sraffa 1960, p. 85) of switches, as one runs down the envelope, for if only two methods are known in each of 1000 industries, there result 2^{1000} wage curves. Many of them might be inferior so that they would not appear on the envelope, like wage curve $w_4(r)$ in fig. 3, but many might appear several times, like $w_1(r)$ in fig. 3, so that "the number of switchpoints" on the envelope could be "at least of the same order of magnitude as the number of wage curves" (Schefold 1997 [1979], p. 279). I could believe this (not alone – I remained unopposed in many presentations of this argument), because I also believed that any two wage curves might cross several times.

There seemed to be no compromise between the idea of an envelope involving a very large number of wage curves and Robinson's postulate that one technique would be the best, independently of distribution. But we shall show that the picture may change drastically, if we admit that the wage curves are nearly straight lines as many empirical investigations by Anwar Shaik and others on the basis of input-output analyses have shown (see Mariolis and Tsoulfidis 2010, and Han and Schefold 2006, with the references mentioned in these papers).

The empirical investigation became possible as a result of turning to input-output analysis. In fact, to rely on input-output analysis and not on set-theoretical approaches for the representation of the spectrum of techniques was a paradigm shift, as we shall argue at the beginning of section 3. Han and Schefold (2006) extended the approach to comparisons of 496 = 32 · 31/2 different envelopes of wage curves, resulting from considering pairs combined out of 32 different input-output tables; each pair represented a spectrum of techniques and gave rise to one of the 496 envelopes. We found one case of reswitching. About 3,5% of the 4389 switchpoints exhibited reverse capital deepening or another paradox of capital theory. The vast majority (more than 95%) of switchpoints were of the neoclassical type, with the intensity of capital falling as one moved across the switchpoint with a rise in the rate of profit both at the macroeconomic and the sectoral level. This investigation certainly had its difficulties; the ones I myself regard as most important are discussed in the paper itself – the experiment should be repeated by others. But it had the advantage of giving a precise answer on the basis of numerous data what a low probability of the most important paradox (reverse capital deepening at the aggregate level) means: less than one percent of the switchpoints observed exhibited this phenomenon.

There was another, curious finding. It turned out, with 4389 switchpoints on 496 envelopes, that only about ten wage curves appeared on average on each of the envelopes, although each envelope was derived from two input-output tables, with 33 sectors. The book of blueprints thus consisted in each comparison of 66 methods for 33 industries so that the construction of each envelope involved $2^{33} \approx 10^{10}$ wage curves.

I have since endeavoured to explain theoretically, why the wage curves must be nearly linear and why the paradoxes therefore are rare (Schefold 2008 and Schefold 2011), but I had no explanation for the puzzle why only so few wage curves (about one in 10^{9}) appeared on each of the envelopes.

A model capable of explaining this finding shall here be presented. It seems useful to increase the number of countries compared in order to understand its significance. But, before stating the main assumptions, it also seems useful to discuss the meaning of the comparison of the input-output tables in more detail. Joan Robinson once wrote: 'Nothing could be more idle than to get up an argument about whether reswitching is "likely" to be found in reality' (Robinson 1979, p. 82). She insisted that two economies separated in space or one economy at two different dates had different states of technological knowledge. Hence she thought that there is 'no such phenomenon in real life as accumulation taking place in a given state of technical knowledge'. Her argument has been repeated by Neokeynesians as a criticism of neoclassical theories, but also in order to question the relevance of the neoricardian analysis of capital; hence the necessity to deal with it here. Comparisons of coexisting techniques are made all the time and this suggests that there is something in the idea of 'accumulation in a given state of technical knowledge' (ibid.). Much public interest is focussed on the question of the choice technique in the energy sector. German electricity companies are imitating Danish windmills just as once German railway engineers imitated British railway construction, and in both cases the new method replaced another, which was also known. To use wind power today means to return to an old technique, certainly with modifications, but using old knowledge, foreign knowledge and some new ideas.

Consider a larger geographical region. If there are e.g. ten countries each represented by 100 sectors, and if we assume that the entrepreneurs in any given industry and a given country have some knowledge about the methods employed by their rivals in some industry in the nine other countries, a great deal of international comparisons are made by all the entrepreneurs taken together, in a decentralised fashion, and there will be much striving according to the models set by others.

Of course, the methods cannot always easily be transferred. There are natural differences: the transport sector of Greece (ships) and that of Austria (railways) are different. We abstract from this at present, because there are so many industries where the natural differences are irrelevant, both in manufacturing and in the services. There are institutional differences. Trains cannot be as fast in Germany as in France, because German federalism indirectly prescribes that trains stop more often in smaller towns. The institutions can be transferred, if people are willing, but only slowly. The same is true for fixed capital. Formally, fixed capital can be reduced to integrated systems, which are akin to circulating capital systems (Schefold 1989 [1971]); it then takes a certain number of years to build a stock of machines of balanced age composition. Or fixed capital can be treated in the manner of Leontief, and a new stock has to be built up, if the method is copied in one country from another. Both approaches demonstrate that the transfer of methods involving fixed capital is slower than if only circulating capital is involved, but it remains feasible. Leontief's approach is more amenable to empirical analysis, since data for the stock matrices are more readily available than the data which are required, if one wishes to follow von Neumann's and Sraffa's joint production approach to fixed capital (Schefold 2012). Results by Mariolis and Tsoulfidis (2010) seem to show that the wage curves are closer to linearity, if fixed capital according to Leontief is taken into account, than if all capital is circulating.

But we here stick to circulating capital. It can further legitimately be objected against the idea of transfer that there are linkages between industries. If a country wants to follow the example given by another and adopt modern windmills for electricity production, it will also have to adopt methods to store energy, e.g. by pumping water to elevated artificial lakes, in order to use this water reserve for electricity generation when the wind does not blow. Because of the decentralised character of wind power generation, it will need a large grid, connecting areas where the wind blows strongly and regularly with centres of consumption. A country relying on nuclear energy needs a different smaller grid, but it will need access to reprocessing plants. But it does not matter much for the other industrial sectors in which way the electricity is produced, provided only that that it becomes available. Hence these linkages do not concern the entire economy, and the most important of them will have been taken into account by aggregation. As the example of electricity generation shows, each of 100 sectors in an input-output table comprises a multitude of connected activities. I do not deny that linkages embracing the economy as a whole may exist. The possibly most important example would be represented by national innovation systems, which contain manifold institutions that connect public and private research. But we exclude such linkages in this paper, except for a brief consideration in section 3.

The products of the sectors are in general not strictly homogenous (except for electricity and a few others), but the success of the classification of sectors in international comparisons of input-output analysis is proof that the homogeneity postulate is fulfilled sufficiently well for empirical and analytical purposes. How the input-output specialists do this is not our concern here, although their prior aggregation of firms to sectors may hide extreme variations of capital -labour ratios between industries.

Finally, we have to be aware that countries, and industries within countries, are advanced in different degrees; the diffusion of known techniques then almost always is associated with some improvement. Progress and the transmission of given knowledge tend to be linked. But this turns out to be a reason why it is interesting to compare the input-output tables of countries. It is true that it seems paradoxical at first sight to take as a book of blueprints the input-output tables of different countries, for if techniques are mobile: why are they different in different countries? Conversely: if techniques are different, that seems to be proof that they are not mobile. What sense does it then make to compare them and to seek an envelope? If we were in a stationary state, with no technical progress, with capital perfectly mobile and with nor natural or institutional advantages of individual countries, we should in fact expect that the most profitable techniques would have been adopted in each country so that all would use the same technique at the same rates of profits and wages in competitive conditions.

But only the rate of profit is fairly quick to adapt because of the mobility of financial capital; the methods are relatively inert and move in conjunction with progress. The envelope, which can be derived from such a book of blueprints, thus indicates the technique towards which the entrepreneurs should look in each country; at the same time, they estimate what progress could add to the productivity gain resulting from mere imitation. The knowledge both of what the coefficients of the tables indicate as known techniques and what improvements are likely to be feasible is dispersed. External effects in networks, communication among entrepreneurs and engineers and the flexibility of institutions will influence the outcome.

The process of imitation among developed countries thus achieves less than the envelope indicates, insofar as so many obstacles have to be overcome in copying the methods of others, but it achieves more, insofar as progress can be expected to accompany imitation. It may be a big problem for developed countries to identify best-practice techniques, which are constantly evolving. It is a lesser problem for backward countries since it does not matter so much whether they take the really best techniques for their target; the second or third best may still constitute a big advance relative to the position in which they are. Even the planned economy of the Soviet Union was able to move forward quickly, when it was very backward, but it got stuck when quality began to matter more. To catch up becomes the more difficult, the closer one is to the top, and a real overtaking, with a new country taking world leadership in technology, has occurred only a few times in history.

With this interpretation in mind, we return to the formal comparison of wage curves, thought to be derived from input-output tables of actual economies. I present a summary of the theoretical arguments why we may expect them to be nearly linear, in accordance with the empirical results referred to above.

2. Nearly linear wage curves

As usual, prices of the system for a given technique (one method in each industry) follow from

$$(1+r)\mathbf{A}p + w\mathbf{l} = \mathbf{p},$$

$\mathbf{A} = (a_{ij})$; $i, j = 1,...,n$; input-output coefficients, $\mathbf{l} = (l_i)$ labour vector, $\mathbf{p}$ normal prices, w wage rate, r rate of profit. Prices are normalised by means of a numéraire vector $\mathbf{d} = (d_1,...d_n)$, $\mathbf{dp} = 1$, where $\mathbf{A} \geq 0$, $\mathbf{d} > 0$, $\mathbf{A}$ indecomposable and productive. Prices in terms of the wage rate

$$\hat{\mathbf{p}} = \mathbf{p}/w = \left(\mathbf{I} - (1+r)\mathbf{A}\right)^{-1}\mathbf{l} > 0$$

rise monotonically from $\hat{\mathbf{p}}(-1) = \mathbf{l}$ via $\hat{\mathbf{p}}(0) = \mathbf{u}$ (labour values) to infinity at $r = R > 0$ (R maximum rate of profit of this system). Hence the wage rate $w(r)$ follows from $1 = \mathbf{dp} = \mathbf{d}\hat{\mathbf{p}}\,w$; $w = 1/\mathbf{d}\hat{\mathbf{p}}(r)$ falls monotonically, $w(r) > 0$; $0 \leq r \leq R$; $w(R) = 0$. Suppose that $\mathbf{d}$, the numéraire, is also the net product of the economy, produced at activity levels $\mathbf{q}$, $\mathbf{q}(\mathbf{I} - \mathbf{A}) = \mathbf{d}$, so that output per man employed $y = \mathbf{dp}/\mathbf{ql} = 1/\mathbf{ql} = w(0)$, $\mathbf{ql}$ employment, is constant in the stationary state. Capital per head $k = \mathbf{qAp}/\mathbf{ql}$ follows from $y = rk + w$, $k = (y - w)/r$; it varies with r along the wage curve, unless the wage curve is linear; one can read k off the wage curve; $k = tg\alpha = (w_1 - w_0)/r$, as at P_0 in fig. 3. In the neoclassical case of fig. 2, each wage curve with $w_1(0) > w_2(0) > w_3(0) > w_4(0)$ is associated with a unique capital-intensity $k_1 > k_2 > k_3 > k_4$.

Consider the spectrum of eigenvalues of matrix $\mathbf{A}$. If we exclude imprimtive matrices, which are of interest only as special cases (see Schefold 2008), $\mathbf{A}$ has a unique Frobenius eigenvalue μ_1, $0 < \mu_1 < 1$, such that all other eigenvalues $\mu_2,...,\mu_n$ are smaller in modulus; they may be ordered $\mu_1 > |\mu_2| \geq |\mu_3| \geq ... \geq |\mu_n| \geq 0$. It would be possible to include eigenvalues that are semi-simple roots of the characteristic equation, using the approach of Schefold 1989 [1971], but we exclude them in order to keep the elegance of the formulae (semi-simple roots are not generic anyway). Then we obtain the otherwise perfectly general expression, using the same approach as in Schefold 1989 [1971], with $\mathbf{q}_i$, $\mathbf{x}_i$ being the left-hand and right-hand eigenvectors of $\mathbf{A}$; $\mathbf{q}_i\mathbf{A} = \mu_i\mathbf{q}_i$, $\mathbf{A}\mathbf{x}_i = \mu_i\mathbf{x}_i$; $i = 1,...,n$; and $\rho = 1 + r$, $\mathbf{q}_i(\mathbf{I} - \rho\mathbf{A}) = (1 - \rho\mu_i)\mathbf{q}_i$:

$$\begin{aligned} 1/w(r) &= \mathbf{d}\hat{\mathbf{p}} = (\mathbf{q}_1 + ... + \mathbf{q}_n)(\mathbf{I} - \rho\mathbf{A})^{-1}\mathbf{l} \\ &= \sum_{i=1}^{n} \frac{\mathbf{q}_i\mathbf{l}}{1 - \rho\mu_i} = \sum_{i=1}^{n} \frac{\mathbf{q}_i\mathbf{x}_i}{1 - \rho\mu_i} \end{aligned} \quad . \qquad (1)$$

Here we have introduced a representation of $\mathbf{d}$ and $\mathbf{l}$ as linear combinations of the $\mathbf{q}_i$, $\mathbf{x}_i$; $i = 1,...,n$; respectively, with the 'strong' normalisation $\mathbf{d} = \mathbf{q}_1 + ... + \mathbf{q}_n$, $\mathbf{l} = \mathbf{x}_1 + ... + \mathbf{x}_n$ (the eigenvectors are so normalised that the coefficients in the linear combinations are all equal to one). Further, we have used that $\mathbf{q}_i\mathbf{x}_j = 0$ for $i \neq j$ since eigenvectors pertaining to different eigenvalues are orthogonal.

Formula (1) is general, setting aside the technical complications which might spring from the non-generic semi-simple roots. All nominators in (1) are positive for $1 \leq \rho < 1/\mu_1$ for those eigenvalues μ_i, $i = 2,...,n$, which are real. If μ_i is not real, there will be another corresponding conjugate complex root so that the sum will become real for ρ real. As ρ tends to $1/\mu_1$; $\mu_1 = 1/(1+R_1)$; R_1 maximum rate of profit, w tends to zero.

The form of (1) confirms that wage curves can be very complicated, with n being large, but one immediately obtains the following familiar simplifications:

If $\mathbf{d} = \mathbf{q}_1$, $\mathbf{q}_2 = ... = \mathbf{q}_n = 0$, $\mathbf{d} = \mathbf{q}_1$ is proportional (not necessarily equal) to Sraffa's standard commodity and the wage curve becomes linear. We call $\mathbf{q}_1$ the Sraffa vector pertaining to $\mathbf{d}$, even if the $\mathbf{q}_2,...,\mathbf{q}_n$ do not all vanish.

If $\mathbf{l} = \mathbf{x}_1$, $\mathbf{x}_2 = ... = \mathbf{x}_n = 0$, the labour theory of value holds because $\mathbf{l}$ is the right-hand side eigenvector of $\mathbf{A}$ so that the organic compositions and the capital intensities are the same in all sectors. The wage curve is linear. We call $\mathbf{x}_1$ the Marx vector pertaining to $\mathbf{l}$, even if the $\mathbf{x}_2,...,\mathbf{x}_n$ do not all vanish.

Not yet familiar (but compare Schefold 2008 and 2011) is the case $\mu_2 = ... = \mu_n = 0$. The wage curve becomes a hyperbola:

$$1/w = \frac{\mathbf{q}_1\mathbf{x}_1}{1-\rho\mu_1} + \mathbf{q}_2\mathbf{x}_2 + ... + \mathbf{q}_n\mathbf{x}_n . \qquad (2)$$

This case looks at first as if it were only of formal relevance, but it turns out to be of great economic interest. It is discussed with more rigour and with more ramifications in Schefold (2011). Here I hope to provide a useful complement to that exposition by presenting a more intuitive and more concise argument.

If the non-dominant eigenvalues are all strictly equal to zero, $\mathbf{A}$ is a matrix of rank 1, and, being semi-positive and indecomposable, $\mathbf{A}$ must be positive and can be written as $\mathbf{A} = \mathbf{cf}$, where $\mathbf{c}$ is a positive column and $\mathbf{f}$ a positive row.[4] This may seem special, but, for $\mathbf{f} = \mathbf{e} = (1,...,1)$, $\mathbf{A}$ is the determinate limit case of random matrices, discussed in Schefold (2011). This looks even more special, but random matrices can be regarded as perturbations of $\mathbf{A} = \mathbf{ce}$ such that the individual coefficients on any row can vary a great deal. On the other hand, the condition that $\mu_2 = ... = \mu_n = 0$ is relaxed: the $\mu_2,...,\mu_n$ are only required to be small (in modulus). The main result is as follows: *It can be proved that the non-dominant eigenvalues tend to disappear for large random matrices, essentially defined by the condition that the coefficients on each row are i.i.d. around a mean specific for the row*. The coefficients on each row are thus distributed with a certain variance as is explained in more detail in Schefold (2011), with references to the relevant mathematical literature. The distribution does not exclude small or zero coefficients, but it is such that different linear combinations of many rows tend to be proportional. Any two given rows may be quite different, but, for large matrices and combinations of many rows, near-proportionality obtains.

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4 It is easy to prove that if $\mathbf{A} \geq 0, \mu_2 = ... = \mu_n = 0$, if and only if $\mathbf{A} = \mathbf{cf}$, $\mathbf{c} > 0, \mathbf{f} > 0$. Note moreover that, if $\mathbf{A}$ is given, $rk\mathbf{A} = 1$, $\mathbf{c}$ and $\mathbf{f}$ are each determined up to a scalar factor, and these factors must be inverse to each other.

The non-dominant eigenvalues of large random matrices only tend to disappear, they are not exactly zero, as stated. Because of the mathematical difficulties associated with the analysis of large random matrices, it is convenient to work with a deterministic analogue, i.e. non-negative matrices, for which all rows are exactly proportionate and which therefore have the property that the non-dominant eigenvalues are strictly equal to zero. Such matrices, which can be written as $\mathbf{A} = \mathbf{cf}$, are necessarily positive, if they are semi-positive. They are artificial constructs, introduced here only in order to visualise some properties of random matrices, which are more realistic.

Another, though mathematically less rigorous, way to describe random matrices is to describe them as perturbations of the elements of matrix $\mathbf{A} = \mathbf{ce}$ – a perturbation, which can be large enough to introduce individual zero coefficients among the elements of $\mathbf{A}$. Now we know that the non-dominant eigenvalues disappear also if $\mathbf{A} = \mathbf{cf}$, $\mathbf{f} > 0$, but $\mathbf{f} \neq \mathbf{e}$. It is clear, for reasons of continuity, that the elements of $\mathbf{A} = \mathbf{cf}$ can be perturbed in such a way that the moduli of $\mu_2,...,\mu_n$ will remain small, as in the case of random matrices. It is not known how far these perturbations may go – a general limit theorem, analogous to that about random matrices of the type $\mathbf{A} = \mathbf{cf}$, with perturbations obeying certain constraints, is not available, to the best of my knowledge. The mathematical theory behind such a theorem would probably be quite difficult, considering how difficult the theorems about random matrices are. But it is clear that non-dominant eigenvalues may be small, even if the distribution of the elements of the input matrix is not i.i.d. The conclusion is simply that (2) is approximately true not only for random matrices, but more generally for large matrices which are random perturbations (not necessarily i.i.d.) of matrices of the form $\mathbf{A} = \mathbf{cf}$. The extent of the admissible perturbations is known as a set of sufficient conditions for $\mathbf{f} = \mathbf{e}$ (random matrices), but not yet in general.

Beginning with an extreme case, we assume that $f_1 > ... > f_n$ and $c_1 > ... > c_n$. $\mathbf{A} = \mathbf{cf}$ then is a technique where commodities $1, 2, ..., n$ are (apart from perturbations, which may be introduced) of the same declining importance in all industries (relative to the unit output of the commodity), and where the industries are hierarchically ordered relative to the unit level of activity, as in the traditional image of the industrial era where e.g. steel was the most important industry (c_1 large, enlarging all $c_1 f_i$) and steel was the most important input in other industries (f_1 large, enlarging all $c_i f_1$), and coal came second, and perhaps corn production third. The ordering is compatible with limited exceptions (because of the admissible perturbations). We call such systems hierarchic. A modern example could be an economy in which information technologies are the leading sector and play a role analogous to steel in the more traditional industrial economies. One might think that hierarchic systems were general among systems of the form $\mathbf{A} = \mathbf{cf}$, for we can always order the sectors so that $c_1 > ... > c_n$. But then we simultaneously define a reordering of the components of $\mathbf{f}$, since the permutations of rows and columns must be simultaneous in single product systems, if the output matrix is to remain the diagonal (unit) matrix. The conclusion therefore is, for (2) to hold strictly or approximately, linear combinations of rows of the system must be proportional on average. Individual rows and coefficients on each row may deviate from the average to some extent, which can be defined exactly in the case of random matrices, while the general mathematical theory has yet to be worked out. In other words, the distribution of the elements on the rows may be i.i.d., or there may be another distribution; an extremely asymmetric distribution obtains, if the system is hierarchic. Hierarchic systems are thus interesting as a limit case, which is intriguing, since ideas of leading sectors and followers recur in the history of economic thought.

After this long, but necessary, digression, we return to the analysis of wage curves of systems which, for whatever reason, being random or not, have small non-dominant eigenvalues so that (2) holds approximately. Now it turns out that much less is needed than that the numéraire be equal to Sraffa's standard commodity or that the labour theory of value be valid to obtain a linear wage curve, if the matrix of the system is of rank one. Consider the vector of deviations $\mathbf{m}$ of the numéraire vector $\mathbf{d}$ from the Sraffa vector $\mathbf{q}_1$

$$\mathbf{m} = \mathbf{d} - \mathbf{q}_1 = \mathbf{q}_2 + \ldots + \mathbf{q}_n$$

and the vector of deviations $\mathbf{v}$ of the labour vector $\mathbf{l}$ from the Marx vector $\mathbf{x}_1$

$$\mathbf{v} = \mathbf{l} - \mathbf{x}_1 = \mathbf{x}_2 + \ldots + \mathbf{x}_n .$$

Let $\overline{m}$ designate the mean of the components of $\mathbf{m}$ and $\overline{v}$ the mean of the components of $\mathbf{v}$. If $\overline{m} = 0$, the deviations of the numéraire from the (standard) Sraffa vector are zero on average, and if $\overline{v} = 0$ the analogue holds for the labour deviations and one might say loosely, in a Marxian vein, that the labour theory of value holds on average. Now on the one hand, using the orthogonality condition:

$$\mathbf{m}\mathbf{v} = (\mathbf{q}_2 + ... + \mathbf{q}_n)(\mathbf{x}_2 + ... + \mathbf{x}_n) = \mathbf{q}_2\mathbf{x}_2 + ... + \mathbf{q}_n\mathbf{x}_n.$$

On the other hand, one has the known formula for the covariance of coefficients of the deviations (considered as random variables):

$$cov(\mathbf{m}, \mathbf{v}) = (1/n)\mathbf{m}\mathbf{v} - \overline{m}\,\overline{v}.$$

There is no obvious reason for a significant correlation between $\mathbf{m}$ and $\mathbf{v}$. The numéraire vector $\mathbf{d}$ can be chosen arbitrarily, while $\mathbf{l}$ can be assumed to be random for a quite different, independent reason: it reflects technology. Similarly, the random character of given $\mathbf{q}_1$ and $\mathbf{x}_1$ depends on the random character of the system as a technique. We are looking for the theoretical potential causes why empirical wage curves turn out to be nearly linear. The solution is first to assume that $cov(\mathbf{m}, \mathbf{v}) = 0$, so that $\mathbf{m}\mathbf{v} = n\overline{m}\,\overline{v}$ and (2) becomes

$$1/w = \frac{\mathbf{q}_1\mathbf{x}_1}{1 - \rho\mu_1} + n\overline{m}\,\overline{v}. \qquad (3)$$

The wage curve of a system, which is random and/or of the form $\mathbf{A} = \mathbf{c}\mathbf{f}$ with perturbations then is nearly linear, the numéraire deviations are zero on average and/or if labour theory of value holds on average:

$$w = \frac{1 - \rho\mu_1}{\mathbf{q}_1\mathbf{x}_1}. \qquad (4)$$

(4) is Sraffa's wage curve. We have thus made a big theoretical advance relative to a long-standing discussion: We have found that the linear wage curve results not only if one has the standard commodity or if the labour theory of value holds. *It is sufficient that either of these properties holds on average*, more formally, that $\overline{m}=0$ and/or $\overline{v}=0$, provided the system is random, or, more generally, that the non-dominant eigenvalues are small.

The expressions (2) and (3) are important even if linearity does not obtain, because they allow to explain the complications of the wage curves: if a wage curve is not a hyperbola, it must be due to non-dominant eigenvalues which are not zero. The work by Mariolis and Tsoulfidis on actual input-output systems has shown that most but not all eigenvalues are close to zero. If ordered according to the moduli, they seem to fall rapidly towards zero according to an exponential law. This tendency remains to be explained. Meanwhile, we can show how a wage curve with h eigenvalues (including the dominant) of significant modulus and $n-h$ eigenvalues of negligible size can be represented as the hyperbola of the form (2) or (3), with $h-1$ terms superimposed, which cause shifts and wiggles. Extending the idea of the deviations, we define

$$\begin{aligned}\mathbf{m}_h &= \mathbf{d}-(\mathbf{q}_1+\ldots+\mathbf{q}_h)=\mathbf{q}_{h+1}+\ldots+\mathbf{q}_n \\ \mathbf{v}_h &= \mathbf{l}-(\mathbf{x}_1+\ldots+\mathbf{x}_h)=\mathbf{x}_{h+1}+\ldots+\mathbf{x}_n\end{aligned}.$$

Combining conjugate complex solutions, $\mathbf{m}_h$ and $\mathbf{v}_h$ are real, and, assuming zero covariance, in obvious notation $\mathbf{m}_h\mathbf{v}_h = n\overline{m}_h\overline{v}_h$. This yields

$$1/w - \frac{\mathbf{q}_1\mathbf{x}_1}{1-\rho\mu_1} = \sum_{i=2}^{h} \frac{\mathbf{q}_i\mathbf{x}_i}{1-\rho\mu_i} + n\overline{m}_h\overline{v}_h . \quad (5)$$

The right-hand side of (5) contains the terms which may cause deviations form linearity. If one compares the wage curves of many systems, using the same numéraire for all, it cannot be assumed that $\overline{m}_h$ will be very small, since, if one assumes that $\mathbf{d} = \mathbf{x}_1$ for one of those systems, there may be a non-random drift in the transition to the other systems, but $\overline{v}_h$ could be quite small for most systems.[5] Hence we assume that the last term in (5) can be neglected in most cases. The influence of the first $h-1$ terms on the right-hand side of (5) will be small, if the corresponding contributions $\mathbf{q}_i$ to the numéraire deviations and $\mathbf{x}_i$ to the labour value deviations will be small. Their influence will grow as $\rho\mu_i$ approaches one,[6] but it will not become infinite. *Hence the possibility to explain why nearly linear wage curves will be relatively frequent, why strongly curved wage curves with considerable wiggles will be less frequent, and why the deviations from linearity are larger at higher rates of profit, as the empirical wage curves show.*

The question now is how many wage curves will make it and appear on the envelope. If that number is small, the envelope can be expected to be composed

5 Theoretically, $\overline{v}_h$ could be large, of course. But it, if the systems are, technically speaking, random also with respect to the labour coefficients , $\overline{v}_h$ must be small, and this is our suggested explanation of the fact, why empirical wage curves in a spectrum of techniques seem to be sufficiently linear to cross only very rarely more than once.

6 Assuming $\mu_i > \mu_1 > 0$. Somewhat different results are obtained, if $\mu_i < -\mu_1 < 0$ or for conjugate complex μ_i, μ_{i+1}. The reader is invited to work out these cases for himself.

mainly of nearly linear wage curves, because *the less frequent outliers remain below*.[7] The main justification for this procedure perhaps bears repetition: We do not postulate a general new theory of nearly linear wage curves, but we propose to explain the empirical finding that wage curves are nearly linear in the relevant range, and for this explanation we do not postulate that input-output tables are generally random or that the labour theory of value holds generally on average, but that these properties hold in combination to a sufficient degree. And who could deny that there is at least some randomness in the emergence of methods of production?

We thus involve a combination of properties to argue that the wage curves encountered on the envelope will tend to be nearly linear. By implication, the amount of capital per head ‘demanded' at each rate of profit will tend to fall, as the rate of profit rises. But will the 'supply' of capital per head fall accordingly as the transition is made from one technique to another at any switchpoint? We must be brief on this point. Since technical change is piecemeal (Han and Schefold 2006), each transition requires the replacement of one and only one method of production by another in one industry, say the first. In the usual neoclassical perspective, the amount of capital is kept constant in the transition and more labour is employed; hence the intensity of capital falls. The capital used with the technique on the left of the switchpoint can be transferred into an equal amount of capital to be used with the technique on the right of the switchpoint. The transition to a newly invented technique (to a higher wage curve) would have required acts of saving and

7 The conclusion is obvious, as long as one has no reason to suppose that the outliers are on average technically superior.

investment, but mere substitution does not need that: the means of production are transformed, that is: the means of production of the first process used on the left of the switchpoint are sold, and with the proceeds the means of production needed on the right of the switchpoint can be bought. But how is this possible, if the means of production are industry-specific? Think of a 'quasi-classical' example: the first industry produces cloth, the method employed on the left uses machines driven by steam engines, the method employed on the right uses hand-driven looms and employs more workers: who wants to buy the machines, if the wage rate falls? Who will have produced the looms? Why the equality of value? Clearly, the machines would have to be written off, as if the Luddites had won, and new investment, financed by saving, would be necessary to buy the looms. Hence we would have to argue in terms of innovation instead of talking about ‘substitution'. This transition would be more plausible as a movement from right to left: the intensity of capital increases with growth. The neoclassical vision of the transition as substitution cannot generally hold, as the quasi-classical example demonstrates, but the neoclassical conception of the 'supply of capital' *is* consistent under the following restrictive conditions: if the means of production are not specific for the industry, those used on the left of the switchpoint can be sold to other industries, and a somewhat different 'combination' of means of production ('factors') can be bought; activity levels will adapt. The transition can be short, if only circulating capital is concerned, but it takes longer with fixed capital. Pure theory avoids the consideration of the transition and is content with the comparison of steady states. In either case, the wage curves will be nearly linear, if the conditions for the averages derived in this section hold to a sufficient degree. More precisely: industries $2,...,n$ do not change and industry 1 must, apart from perturbations, be equal to a linear combination of the other industries, if we are dealing with random systems. *Hence the change of the means of production in the transition must not be larger than the admissible perturbations*. Moreover, the labour theory of value should hold on average. The conditions for the neoclassical theory to hold are essentially the same, both as regards 'demand' and 'supply' of 'capital'.

3. The lens of wage curves and its envelope

The question of whether reswitching is more than a fluke was first approached in terms of set theory. It was shown (Schefold 1976a) that the set of potential methods engendering wage curves that intersect twice with the non-linear wage curve of a given system is not of measure zero in the set of potential methods engendering wage curves that interact at least once with the wage curve of the given system. But the economic content of this concept of measurement was problematic. How densely populated is the continuous space of potential methods with discrete methods that can actually be used? D'Ippolito, Petri, Salvadori, Steedman (see Petri 2010) and others have discussed this with interesting results but the measurement problem has remained. This is why I prefer to start here with a book of blueprints that is thought to result from the comparison of input-output tables. The empirical turn thus made possible represents a new paradigm, prepared by the many articles dedicated to the empirical analysis of wage curves. The methods are given and can be now counted. We thus bridge the gap between theory as a thought experiment and actual measurement, and we can approach the problem of the likelihood of the appearance of the paradoxes by combinatorial methods.

Let therefore k tables for k countries, each with n sectors, be given. We assume that there are no links between methods other than those within sectors. We assume away natural obstacles to the transfer of methods and suppose that, though that may take time, methods can be transferred with the associated institutional changes. We keep in mind that the concept of transition between these techniques is problematic for various reasons, especially because, whenever entrepreneurs strive to replace method α by β, which seems more profitable, they will get new ideas and end up with a method γ, of which it is then a question whether it still resembles β or whether it looks like an outgrowth of α. This is true, but the wage curves and their envelopes remain important theoretical tools.

Given the set of blueprints, we obtain $s = k^n$ wage curves $w_\sigma(r)$; $\sigma = 1,...,s$. We assume them to be strictly linear, to begin with, as will be the case if all techniques are of the form $\mathbf{A}^{(\sigma)} = \mathbf{c}^{(\sigma)}\mathbf{f}$, all indecomposable, with $\mathbf{f}$ as common numéraire; $\sigma = 1,...,s$. We call this the straight lines case. Strictly speaking, two wage curves cannot have a common switch point on the envelope, if they are straight lines, except for one special case (Schefold 2008), but it is almost obvious that we can disregard this problem here.[8] All $w_\sigma(-1) > 0$, since $\mathbf{l}^{(\sigma)} > 0$, but we do not necessarily

[8] To begin with the exception: Let $w_1(r)$ be linear because the labour theory of value holds and assume that another system with wage curve $w_2(r)$ becomes dominant at some rate of profit r_1 which results from the substitution of another method in one of the industries of the first system. Now linearise the second wage curve by taking the standard commodity of the second system as the common numéraire; w_1 will remain linear. But if a third technique appears, with a second switchpoint on the envelope, it cannot in general have equal organic composition of capital in all

have $w_\sigma(0) > 0$. But we can, without loss of generality, assume an ordering such that $w_1(0) > w_2(0) > ... > w_s(0)$. The maximum rates of profit $R_1,...,R_s$ will appear in a different ordering $R_{\sigma_1} > ... > R_{\sigma_s}$, where $(\sigma_1,...,\sigma_s)$ is a permutation of $(1,2,...,s)$. If technique σ is productive, we have $R_\sigma > 0$, but even if it is not, we have $1 + R_\sigma > 0$, hence $R_\sigma > -1$, since $\mathbf{A}^{(\sigma)} \geq 0$. Our s wage curves, s being a large number, will fill a concave lens with $w_{\max}(-1) \geq w_\sigma(-1) \geq w_{\min}(-1)$ and $R_{\max} \geq R_\sigma \geq R_{\min}$, $\sigma = 1,...,s$ as in fig. 4. There is an upper and a lower envelope for the lens; the envelopes will look smooth (although they are composed of a finite number of straight lines) if many wage curves appear on them. But will this be the case?

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industries and the numéraire has already been fixed so that $w_3(r)$ cannot be strictly linear. The difficulty does not matter, because we are her really dealing with quasi-linear wage curves, i.e. wage curves that are nearly linear in the relevant range (usually well below the maximum rate of profit). If a system is large and its wage curve is nearly linear, and if one method of production is replaced, the resulting new wage curve will also tend to be nearly linear for reasons of continuity. In order to simplify the analysis which follows, we assume strict linearity, but it would do to assume that the curvature of wage curves is such that only two of them intersect at most once, in accordance with the empirical envelopes analysed in Han and Schefold (2006), where wage curves with a switchpoint on the envelope had at least one other intersection in common in less than two percent of the more than 4000 cases.

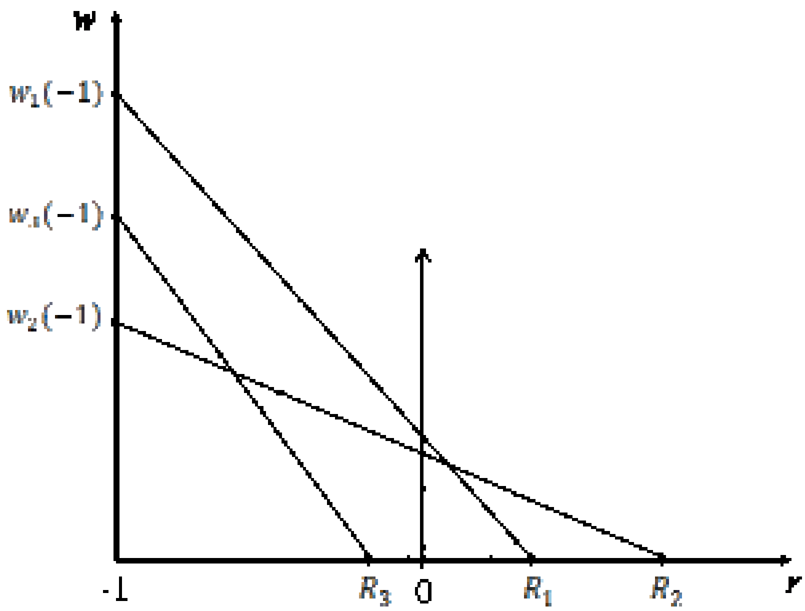

Fig. 4: The lens composed of straight wage curves between two concave envelopes. Ordering of the curves according to the level of the wage at $r = 0$.

A large number of wage curves, namely s, will be found within the lens; only three have been drawn. Observe that the lens will look more like a triangle, if $R_{\max} - R_{\min}$ is small and $w_{\max}(-1) - w_{\min}(-1)$ large. This would be the image corresponding to Kaldor's stylised facts: the capital-output ratio, represented by the inverse of the maximum rate of profit, would stay nearly constant and the capital-labour ratio would rise as one ascended the ladder of wage curves with rising productivity, in a temporal sequence (not in one given period, but in a state of rising knowledge).

What changes, if the straight lines are replaced by monotonically falling curves? The envelope would still be monotonically falling, but the lens would not necessarily be concave. The wage curves of non-productive techniques would still be not positive at positive rates of profit, like $w_3(r)$ in fig. 4. The deviations of the wage curves from straight lines would be most pronounced near the maximum rates of profit, according to the analysis of section 2 above, hence the Kaldorian stylised facts would have to be questioned; the problems of capital theory affect neokeynesian as well as neoclassical economics.

We return to the assumption of straight wage curves in order to address our main theme. We simplify by assuming that all techniques are indecomposable and productive. We number the wage curves so that $w_1(0) > w_2(0) > ... > w_s(0)$, and we make the decisive equal-probability assumption: in the ordering of the maximum rates of profit $R_{\sigma_1} > ... > R_{\sigma_s}$, all permutations $(\sigma_1,...,\sigma_s)$ of $(1,...,s)$ are equally likely. For why should, given such a large number of possibilities, anything else be assumed? If w_σ is relatively small so that $w_\sigma(r)$ represents a 'bad' technique for low rates of profit: why should technique σ suddenly be 'good' at high rates? A neoclassical economist might postulate that a low $w_\sigma(0)$ should be compensated by a high R_σ, but this is justified only as an observation of what perhaps happens on the upper envelope of the wage curves as a result of optimisation. If $w_\sigma(0)$ is low and σ has been chosen at random, R_σ will be random and thus may be high or low. Each technique σ results from the arbitrary combination of n methods, each taken arbitrarily from k tables. The quality of the technique, characterised in the linear case by $w_\sigma(0)$ and R_σ, is unknown, and only optimisation leads to a subset of techniques where the trade-off, high $w_\sigma(0)$ implies low R_σ and *vice versa*, may be visible.

The referee suggested an argument which represents the opposite of the neoclassical supposition: Since some techniques will plausibly embody more technical progress than others, it would seem more likely that a higher maximum wage will tend to be associated with higher efficiency in general and therefore with a higher maximum rate of profit, too. The argument seems to rely on linkages between all sectors, which we had excluded, assuming that linkages hold only within sectors. If, for the sake of the argument, we now admit the general linkages, related to progress, we do not necessarily arrive at the conclusion suggested by the referee. Research for innovation often is at least subjectively directed at either preferentially saving labour or certain means of production. One can also recall the different forms of technical progress with mechanisation and saving of labour being fostered by objective class struggle in Marx (Schefold 1976). They imply a tendency to raise $w(0)$ at the expense of reducing R. But the Marxian argument was strong only as long as it concerned a leading sector (textiles). A more important counterargument, in my view, is the following: Research in any sector takes its direction in a given environment, i. e. given the methods of production and the consequent prices in other sectors, but the overall linkages are too weak to count. The spectrum of techniques consists of all $s = k^n$ methods. A successful national innovation system may render many sectors effective, but combining with methods from other countries can still increase efficiency, even in the country which leads in most, but not all, sectors. Even Mephisto will have difficulties to visualise all possible combinations, and the technique appearing on the theoretical frontier are not likely to be techniques adopted by any of the k countries (a point to which we shall return in the end). Most systems consist of methods of which only a small number coexist in any one country. There are k systems (the actual ones), where all methods coexist. There are $kn(k-1)$ systems, where all but one methods belong to one actual country, and so on. What can we say about the efficiency of the many systems, the methods of which are a combination of methods from many countries? Unsurmountable ignorance compels us to maintain the equal probability assumption. Hence the randomness of the process in which the actual market tends to approximate the results of an ideal market.

If we make the assumption of equal probability, together with the other, more innocent ones, which have been stated, we get at once a definite probability for the Joan Robinson-case of fig. 1 (one wage curve constitutes the envelope). Since $w_\sigma(0)$ is on the envelope by assumption about the ordering, $w_1(r)$ must be the envelope, and we must have $R_1 > R_\sigma$, $\sigma = 2,...,s$. Since R_1 could *a priori* have been in any of the s positions with $R_{\sigma_1} > ... > R_{\sigma_s}$ the probability is $1/s$. If $k = 10$ and $n = 100$, $1/s = 10^{-100}$, a very low probability. In the empirical analysis by Han and Schefold (2006), 496 envelopes of wage curves were analysed. Since $k = 2$ and $n = 33$, $s = 2^{33} \approx 10^{10}$, the occurrence of the Robinson case could not be expected. In fact, the minimum number of wage curves encountered on any envelope was 3.

Next consider the pure neoclassical case where the order of the maximum rates of profit R_σ is *exactly* inverse to that of the $w_\sigma(0)$, as in fig. 2. $R_1 < ... < R_s$ is one permutation in $s!$ possible permutations of $R_1,...,R_s$, hence a probability so small as to be neglected, and the pure neoclassical ordering was, of course, not observed in the investigation by Han and Schefold.

If the ordering of $R_1,...,R_s$ is not exactly inverse to that of $w_1(0),...,w_s(0)$, not all wage curves will appear on the envelope, and the wage curves appearing on the envelope, if sufficiently numerous, could still constitute something like a neoclassical production function, with a certain number of inefficient techniques with wage curves totally below the envelope left out. How many wage curves do we have to expect to appear? We derive an upper bound for this expectation.

The probability that $w_1(r)$ is on the envelope equals one, since $w_1(0) > w_\sigma(0)$, $\sigma = 2,...,s$. It is clear that at least one technique must be on the envelope.

The probability that $w_2(r)$ appears on the envelope equals at most $1/2$, since it is necessary for the appearance that $R_2 > R_1$, and this is one of two equally probable cases: $R_1 > R_2$ and $R_2 > R_1$.

The probability that $w_\sigma(r)$ appears on the envelope equals at most $1/\sigma$, since it is necessary that $R_\sigma > R_\tau$, $\tau = 1,...,\sigma - 1$ (R_σ must be in one of σ equally likely positions). It is intuitive that the probability for $w_\sigma(r)$ to appear on the envelope diminishes as $w_\sigma(0)$ diminishes.

The expected upper bound for the total number of wage curves, say $\Omega(s)$, *on the envelope is the sum of the probabilities of the cases*[9]*, hence* $1 + 1/2 + ... + 1/s$ *which tends to* $\ln s$, *as* s *increases*. About ten wage curves were found on the envelopes on average in Han and Schefold (2006), but $\ln(2^{33}) = 33 \cdot \ln 2 \approx 22.8$. The formula $\Omega(s) = \ln s$ represents in fact an upper bound, for, depending on the spacing of the

9 Intuitively: imagine that you are a beggar walking in the streets of a city, and in each street σ you are given 1 € with probability p_σ . Hence, if you walk in the streets $1,...,s$, you expect to receive $(p_1 + ... + p_s)$ €. Now imagine that you are walking down the envelope. The expectation of the number of wage curves is $1 + 1/2 + ... + 1/s \approx \ln(s)$.

$w_\sigma(0)$ and the R_σ, some wage curves and corresponding switchpoints may get *dominated*, as is illustrated in fig. 5, where w_2 is not on the envelope (although $R_2 > R_1$), if the third wage curve is, given $R_3 > R_2$, defined by $\tilde{w}_3(0)$, whereas w_2 is on the envelope, if $w_3(0)$ is sufficiently small.

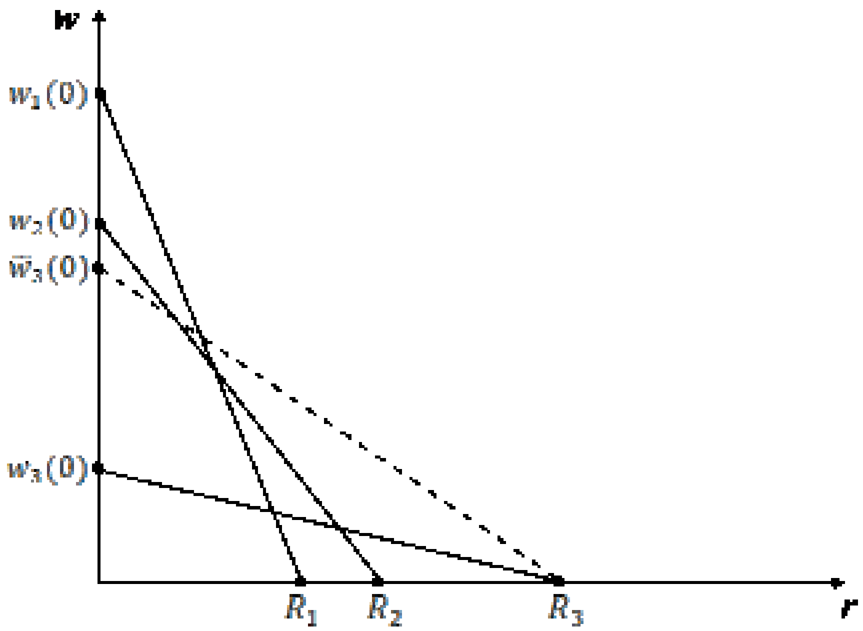

Fig. 5: Three wage curves would be on the envelope with appropriate spacing of the $w_\sigma(0)$, R_σ, and $\tilde{w}_3(0)$ sufficiently small, but only two appear, if w_3 starts at $\tilde{w}_3(0)$, $\tilde{w}_3$ dotted wage curve.

Even if we neglect this domination effect[10], the number of wage curves, which appear on the envelope is surprisingly small. The share of wage curves appearing on the envelope, say Θ, is for the upper bound

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[10] Is there a sufficient condition to exclude the domination effect? I guessed that equal spacing between the $w_1(0),...,w_s(0)$ and between $R_{\sigma_1},...,R_{\sigma_s}$ might suffice, but Christian Bidard gave a numerical example which proves that this is not the case.

$$\Theta = \frac{\ln s}{s}$$

which tends to zero for $s \to \infty$. In fact we have, if again $k = 10$ and $n = 100$, $\ln s = \ln 10^{100} = 100 \cdot \ln 10 \approx 230$ – a surprisingly low number!

And yet something remains of the idea of the surrogate production function in this example. If the greatest of the maximum rates of profit is 100%, say (we are representing circulating capital only so far), each change of the rate of profit by one percentage point induces about two changes of methods, on average, and if real wages are pushed up so that the rate of profit falls by several percentage points, it becomes, in theory, profitable to make several substitutions which raise capital per head.

But to get from this analysis to the elegant properties of production functions with a given and constant elasticity of substitution,[11] problematic additional assumptions would be required. In order to obtain more techniques on the envelope, given k and n, one would have to assume that the likelihood of R_σ being large increased as $w_\sigma(0)$ fell, and the $w_\sigma(0)$ and the R_σ would have to be so spaced as to obtain the curvature of the envelope which would give rise to a Cobb-Douglas or a C.E.S. production function.

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11 A constant elasticity of substitution is not required for the neoclassical theory in its general form, but for most relevant applications in the modern theory of growth.

We thus do not arrive at a full restoration of the production function. Another critical possibility[12] is that the elasticity of substitution of the intensity of capital to the rate of profit might be so low as to be irrelevant: If there are thirty switches, say, in the relevant range of the rate of profit, none of them associated with reverse capital deepening, but causing an overall change of capital per unit of labour of only about five percent, the increase in the demand for labour would be too low, even conceding a given 'supply of capital‘. The argument again turns empirical at this point. Neoclassical authors have sought to render their argument plausible by considering the extreme cases: If wages fell really low, pre-industrial methods of production would again become profitable. Ten workers using the spade for free are cheaper than one, also working for free, but using a tractor. Only capital costs count at $w = 0$, $r = R$; labour costs nothing. The converse case is more difficult and possibly more relevant. Whether unemployment can be removed by lowering wages and mere technical substitution, without stimulating effective demand, in a closed economy, is a somewhat academic question – the main positive effects on employment from lowering wages in practice will come from increases in exports. But what about the ability of the capitalist class as a whole to lower employment, while maintaining production, in reaction to high wage claims? Will an economy with strong trade unions end up with what some call a higher 'natural' rate of unemployment? This question, which plays a central role in the Marxian theory of accumulation (mechanisation as a mean to save labour, resulting periodically in crises), would be formally the opposite of $w = 0$ and $r = R$, not if $r = 0$, but if $r = -1$! We are used to say that wages are their maximum, if the rate of profit is zero, because we cannot really conceive of a negative rate of profit. But, at $r = 0$, only no

12 This was pointed out by the referee. His formulation is being quoted.

profit is earned on the value of capital advanced; capital must still be paid for. By contrast, at $r = R$, there is no expense on wages.[13] If the machines could be used for free and all costs were direct wage costs, as at $r = -1$, indefinite amounts of machines could be used to replace labour; then it is plausible that this would happen. However, $r = -1$ is surely outside the relevant range of the rate of profit (although we found it analytically convenient to include $r = -1$ in Fig. 4 in order to analyse the 'lens'). Whether the elasticity of the intensity to capital is high enough to create unemployment by substituting capital for labour in the relevant range (w high, r low but positive) is again an empirical question. To this extent, the referee is surely right to ask, whether the elasticity of the intensity of capital with respect to distribution is really high enough to justify the explanation of unemployment by high wages in the context of a closed economy.

At this point, the reader, reminded of economic history, will remember other problems of the choice of technique. Even if we stick to the linear wage curves, the suspicion arises that the economy will hover below the surface of the ocean among a multitude of not quite efficient techniques, in accordance with the vision discussed in section 1 of this paper: That diffusion takes time, that the imitation of known technology is mixed up with progress and that such imitation becomes more challenging as one approaches the efficiency frontier. Hence it seems better to work with the apparatus of the wage curves and their envelope, and not with the problematic idealisation, the production function.

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13 If the subsistent wage is not counted among the means of production, as Sraffa (1960) suggests (§ 8).

What remains of this analysis, if the wage curves are not strictly linear? We have argued that the deviations from linearity will only exceptionally be large and that the number of wage curves on the envelope will be relatively small, hence we may expect that the wage curves deviating drastically from linearity even at small rates of profit are likely to be inside the lens of all wage curves. However, more empirical work, involving the comparison of the wage curves derived from different input-output tables, not only the wage curves derived from individual input-output tables, is needed, as well as more theory to explain the curious spectra of eigenvalues of empirical input-output tables, before we can come to safe conclusions. We hope to have opened up a field of research where the existence o the production function and related questions can be discussed by other means than mere *a priori* reasoning.

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