DEMAND-LED GROWTH IN A MULTI-COMMODITY MODEL WITH LEARNING

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Abstract

The paper represents an attempt to shed light on the following question: in the context of demand-led growth, how does learning by agents about the economic system's structure and the determinants of long-run growth affect the long-run dynamics of the economy? Analysis is conducted in terms of an extension of a simplified two-sector model with fixed capital and joint production and autonomous demands used in White (2008). The focus of the analysis is on the impact of learning about two mechanisms in particular: about how the growth of autonomous demand influences growth of the economy as a whole; and about how expectations about growth affect the dynamics of growth. The mechanics of learning are twofold: first, a simple gradient-descent rule, where a key parameter of the investment function relating producers expectations about growth to past growth in their own sector and in the economy are modified in a way which aims to minimize forecast errors; and, second, a more ambitious mechanism whereby producers attempt to uncover aspects of the true relation between past growth rates and expected growth rates. This latter also involves at least some producers making use of a simplified one-sector "model" of the economy. Hence, the paper also provides for some reflection on the adequacy of one-sector models as a means of capturing multi-sectoral macro dynamics. Analysis of the system's dynamics is primarily by means of computer simulation.

Classification codes: 041, B50, E12, E37

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DEMAND-LED GROWTH IN A MULTI-COMMODITY MODEL WITH LEARNING: SOME SIMULATION RESULTS

I Introduction

The present paper is a tentative step towards exploring the significance of learning for the dynamics of growth, where the latter is ultimately driven by demand. Research on demand-led or demand-constrained growth represents a rich undercurrent in the history of twentieth century economics up to the present day (*cf.* Blecker, 2002, Commendatore, 2003, Garegnani and Palumbo, 1998, Palumbo A. and Trezzini A., 2003 and White, 2006 for surveys); a history which has nonetheless been dominated – at least in respect of growth analysis – by a supply-driven approach.

The analysis in the present paper is an attempt to utilize the approach used by Caminati (1998) in relation to Harrod's warranted growth path; but apply it here to a two-sector, fixed capital, demand-led growth model of the type explored in White (2008): an analysis of demand-led growth but one which did not provide for agents to learn. The present paper represents a natural extension of that analysis by allowing for learning. It follows the spirit of Caminati (1998) in contrasting two types of learning: one simplified and which does not attempt to uncover the dynamics of the system; and a more complex learning which seeks a deeper understanding of the system's dynamics.

Since growth is assumed to be demand determined the nature of complex learning is arguably different from what one would expect in a more conventional, supply-driven approach. In the latter case, learning about growth entails agents uncovering the "true" parameters of the economy which are independent of the growth rate of aggregate demand. Where growth is demand-driven, the true or objective dynamics, which learning by agents is designed to provide insight into, are not independent of demand growth which itself depends on expectations about demand growth. Uncovering the true long-run dynamics of demand and thus of output – cannot but involve learning about how agents form expectations about demand on the basis of past demand realisations.

The aim of the paper is a relatively modest one: to focus exclusively on the dynamics of quantities which emerge from multiplier-accelerator interaction in a multi-commodity setting with learning.[1] As such, price dynamics are ignored. Prices are assumed to be fixed at their their long-period configuration; i.e. the configuration which would yield a uniform rate of return across production processes. Though, extremely simplified, the present analysis is intended to serve as a first step or "control" for subsequent analysis of learning in a model which allows for price dynamics. Fluctuations in the growth rates of the autonomous components of demand are also for the most part ignored.

The analysis is developed in terms of a discrete-time, dynamic model with the focus on how agents' realizations as to the nature of the system's dynamics feedback on the system's growth. The key complexity focused upon lies in the determination of investment and more specifically, in the determination of expectations on the part of producers about the growth of demand. It is through this channel and thus via investment decisions that the effects of learning by producers are assumed to impact most on the process of growth.

The discussion takes place in four stages: first, by considering the dynamics of demand purely generated by multiplier-accelerator interaction without any learning (sections II – IV); second, by considering how these dynamics are affected by a simple, mechanical form of learning (section V); third by focusing on how these dynamics in turn are affected if at least some agents seek to learn and model the objective dynamics of demand as an integral part of formulating expectations about demand growth (sections VI - VII); and, finally, by considering the dynamics of demand when the proportion of producers seeking this deeper form of learning is made endogenous (section VIII). Section IX provides some brief concluding comments.

II A two-sector fixed capital model

(i) Production

The basic model which informs the analysis of this paper is that of White (2008). This model is only sparsely set out here. Consider a discrete-time model of an economy with two production processes: one producing a pure consumption good – commodity 1 (not required as a direct input for either production process); the other producing a durable capital good (machine) of varying efficiency – commodity 2. Each commodity is produced in a separate

[1] In this sense it is also close in spirit to Harrod's (1939) analysis, his warranted growth path reflecting a particular case of multiplier accelerator interaction.

industry and each produced by means of a combination of labour and machines (in fixed but different proportions). Machines have a maximum (technical) life of two periods, after which they are disposed of costlessly. One-year old machines are treated as joint products, *à la* Sraffa (1960).[2] There is no other joint production. The model ignores foreign trade as well as the government sector.[3]

New machines are assumed to be of a uniform type across the two production processes; while they vary in efficiency over their life. Thus more labour is required by older machines to produce the same output as new machines. The difference in efficiency also implies differences in desired utilization rates between new and old machines, although this is treated in a very simple linear manner by assuming that utilization on older machines is lower than that on new machines by a fixed proportion.

Regarding the relationship between output and demand, the model assumes a one-period lag between demand and production and thus the need for producers to forecast future demand in the planning of output.[4] With time divided into discrete periods, together with the assumption that demand for each commodity is expressed at each junction of two periods, while production takes place during periods, planning for production in t requires producers to estimate the demand forthcoming at the conclusion of period t.

More precisely, the model assumes that production of commodity i undertaken in period t (Y_{it}) is based on demand realized at the end of period $t\text{-}1$ (D_{it-1}) as well as the expected growth rate of demand between periods t and $t\text{-}1$, $g^{de}{}_{it-1}$, so that

$$Y_{it} = D_{it-1}\left(1 + g^{de}_{it-1}\right) \qquad \text{........(II.1)}$$

The determination of $g^{de}{}_{it-1}$, is taken up in section IV below. Since $D_{it-1} = D_{it} / (1+g^{d}{}_{it})$, where $g^{d}{}_{it}$ is the realized growth rate of demand in sector i between periods t and $t\text{-}1$, expression (II.1) implies that

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[2] In other words, at the end of each year the output of the production process using labour and new machines is a quantity of commodity 1 or 2 plus a quantity of one-year old machines.

[3] Though demand arising from foreign trade and /or government intervention could well provide part of demand which is referred to here as “autonomous”.

[4] This contrasts with the approach in White (2008), where output responds within each period to demand expressed within the same period.

$$Y_{it} = \frac{D_{it}\left(1+g^{de}_{it-1}\right)}{1+g^{d}_{it}} \qquad \text{........(II.2)}$$

(ii) Investment

Over the long-run, capacity (the stock of machines – commodity 2) in each sector is assumed to adjust to demand. More precisely, it is assumed that in making investment decisions at the end of period t producers estimate of demand forthcoming at the end of period $t+1$, and in turn estimate the capacity required assuming a desired utilization rate of $u^{n0}{}_{it+1}$ on newly installed capacity; and thus form an estimate of the extent to which capacity at the end of period t is deficient or excessive. With $D^{e}{}_{it+1}$ representing demand expected in sector i at the end of period $t+1$, producers' desired capacity comprising one-year old machines in $t+1$, which were new in period t, and new machines to be installed for use in $t+1$, would satisfy the expression

$$D^{e}_{it+1} = \left(u^{n}_{i}.M^{0}_{it+1} + \frac{u^{n}_{i}.M^{1}_{it+1}}{(1+\phi)}\right).\beta_i \qquad \text{........(II.3)}$$

where $M^{0}{}_{it+1}$ and $M^{1}{}_{it+1}$ represent respectively for period $t+1$ the planned new and one-period-old (new in period t) machines in sector i, while $u^{n0}i$ and $u^{n1}i$ represent respectively the corresponding normal utilization rates and β_i represents the output capacity of a machine in terms of commodity i.

Since $M^{1}_{it+1} = M^{0}_{it}$ and noting that investment demand I_{it} during period t leads to the installation of an amount of new capacity $M^{0}{}_{it+1}$ [5] for use in $t+1$, then expression (II.3) can be written as

$$D^{e}_{it+1} = \left(u^{n}_{i}.I_{it} + \frac{u^{n}_{i}.M^{0}_{it}}{(1+\phi)}\right).\beta_i \qquad \text{........(II.4)}$$

Demand expected in $t+1$ is assumed to be based an extrapolation of demand observed in period $t-1$, so that

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[5] The implicit simplifying assumption behind the equality of $M^{0}{}_{it+1}$ and I_{it} is that the capacity constraint in the machine-producing industry is non-binding; and thus, effectively, that any excess of demand for new machines at the end of period t over capacity output in this industry can be met out of inventories. The model does take account of a capacity constraint in the production of each commodity to the extent that for simulation purposes (see sections IV-VIII below) utilization cannot be greater than 100%. This will entail some restriction on aggregate demand by restricting employment and thus the wage bill and in turn consumption demand.

$$D^e_{it+1} = D_{it-1}.\left(1+g^{de}_{it}\right)^2 \qquad \text{……..(II.5)}$$

where g^{de}_{it} refers to the expectation held at the end of period t, about the future rate of growth of demand for commodity i.[6] On the basis of expressions (II.4) and (II.5) investment demand in sector i at time t can be written as

$$I_{it} = \frac{D_{it-1}.\left(1+g^{de}_{it}\right)^2.(1+\phi) - u^n_i.M^0_{it}.\beta_i}{u^n_i.\beta_i.(1+\phi)} \qquad i = 1,2 \qquad \text{……..(II.6)}$$[7]

Hence demand for new machines - commodity 2 – at the end of period t is

$$D_{2t} = \sum_{i=1}^{2}\left(\frac{D_{it-1}.\left(1+g^{de}_{it}\right)^2.(1+\phi) - u^n_i.M^0_{it}.\beta_i}{u^n_i.\beta_i.(1+\phi)}\right) + I^{Aut}_t \qquad \text{……..(II.7),}$$

where I^{Aut}_t represents autonomous non-capacity creating demand for commodity 2 expressed during period t.

Dividing through equation (II.7) by D_{2t-1} and noting $M^1_{it+1} = M^0_{it}$ and $M^0_{it} = I_{it-1}$ with $D_{2t-1} = I_{1t-1} + I_{2t-1} + I^{Aut}_{t-1}$, and $D_{12t-1}=D_{1t-1}/D_{2t-1}$, one can arrive at an expression for the growth rate of demand for machines $g^d_{2t} = \left(D_{2t}/D_{2t-1}\right) - 1$ as a function of expected growth rates, relative size of the two sectors, the rate of growth of autonomous demand and the share of autonomous demand in total demand:

$$g^d_{2t} = g_2\left(g^{de}_{it}, D_{12t-1}, ID^{Au}_t, a_t\right) \quad i = 1,2 \qquad \text{……..(II.8)}$$

where ID^{Au}_t is the ratio of autonomous demand for commodity 2 to total demand for commodity 2 at t and a_t is the exogenously given growth rate of autonomous demand I^{Aut} for machines.

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6 The squaring of the expression in brackets in expression (II.5) arises because producers in the machine producing sector could not estimate demand forthcoming at the end of period $t+1$ on the basis of realised demand at the end of period t; since they cannot know this magnitude prior to making their own investment decisions (indeed, the former depends on the latter). For simplicity, we assume this to be the case for investment decisions in both sectors.

7 With regard to the financing of investment, it is assumed for simplicity that finance for investment is available at a rate of interest less than the rate of profit implicit in equations (Appendix 1). Thus the model effectively ignores any financing constraint on investment, so that investment in any sector is wholly governed by the expected growth rate of demand for that sector.

(iii) Consumption

Consumption demand – demand for commodity 1 - depends on income and is expressed with a lag of one period. Demand for commodity 1 at the end of period t can be written as

$$D_{1t} = (1 - s_w)Y_{t-1}^{w} + (1 - s_c).P_{t-1}^{c} + D_{1t}^{Aut} \qquad \text{........(II.9)}$$

with s_w, and s_c the saving propensities of workers and capitalists respectively, Y^{w}_{t-1} the income of workers and P^{c}_{t-1} the profit flow to capitalists. D^{Aut}_{1t} represents autonomous demand for commodity 1. With $s_w > 0$ some part of total profit will accrue to workers, so that income of workers in period t denoted as Y_t^w can be written as

$$Y_t^w = \frac{w}{1 - \pi.s_w}\left(L_{1t} + L_{2t}\right). \qquad \text{........(II.10)}$$ [8]

Here w is the real wage in terms of commodity 1 and π is the rate of profit while the parenthetical term represents the total labour requirement for production in the economy in period t.

The flow of profit available to capitalists for consumption expenditure will be influenced by the size of depreciation allowances; and with durable capital treated as a joint product, these allowances will depend on relative prices, including the prices of used machines, and the rate of profit. Depreciation for sector i in period t, denoted λ_{it}, can be expressed as

$$\lambda_{it} = M_{it}^{0}\left(p_{21t} - p_{it}^{m11}\right) + M_{it}^{1}.p_{it}^{m1} \qquad i = 1,2 \qquad \text{........(II.11)}$$

where p^{m1}_{it} refers to the price of one-year old machines used in sector i in period t relative to the price of commodity 1 (*cf.* Appendix 1). Taking commodity 1 as numeraire, profit flow in period t, denoted Π_t, is given by the sum of profits associated with the use of capacity at different ages, less depreciation allowances for each sector, viz.,

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[8] As noted in White, 2008, the simplifying assumption implicit in expression (II.13) means that the profit income available in period t is based on saving undertaken in period t and that this also really assumes that the income estimate on which workers based their consumption in period t includes the expectation of income (profit/interest) to be received in $t+1$from saving in period t.

It should also be noted here that the treatment of durable capital as a joint product along with the assumption of variable efficiency means that the labour requirement in each sector will depend on the age composition of the stock of machines in use.

$$\Pi_t = \sum_{i=1}^{2}\sum_{j=0}^{1}\left\{\left(p_{i1} - l_{it}^{j}.w\right).Y_{it}^{j} - \lambda_{it}\right\} \qquad \text{........(II.12)}$$

where p_{i1} is the price of commodity i in terms of the numeraire. The profit flow, P^c_t available to capitalists at the end of period t is therefore

$$P_t^c = \Pi_t - \frac{w.\pi.s_w}{1-\pi.s_w}\left(L_{1t} + L_{2t}\right) \qquad \text{........(II.13)}$$

where the term $\frac{w.\pi.s_w}{1-\pi.s_w}\left(L_{1t} + L_{2t}\right)$ represents the profit component of workers' income for period t. A final step in determining consumption demand is to define the growth rate of newly installed capacity in sector i between t and $t\text{-}1$ as

$$g_{it}^{m} = \frac{M_{it}^{0}}{M_{it-1}^{0}} - 1 = \frac{I_{it-1}}{I_{it-2}} - 1 \qquad \text{........(II.14)}$$

Given the relation between output and demand, the relation between utilization of new and old machines and the definition of utilization as the ratio of output to output capacity, expressions (II.9) – (II.14) allow, after some manipulation, one to derive an expression for the rate of growth of demand for the consumption good, analogous to that of (II.8) (commodity 2), so that

$$g_{1t}^{d} = g_1\left(g_{it-1}^{d}, g_{it-1}^{m}, u_{it-1}^{0}, D_{12t-1}, M_{12t-1}^{0}, CD_{t-1}^{Au}\right)\ i = 1,2 \qquad \text{........(II.15)}$$

where M_{12t-1}^{0} represents the ratio of new capacities in the two sectors in periods $t\text{-}1$, while CD^{Au}_{t-1}refers to ratio of autonomous demand for commodity 1 to total demand for commodity 1 at $t\text{-}1$.

Similar manipulation of expression (II.14), allows one to eliminate the D_{it-1}'s and M^0_{it-1}'s and express g^m_{it} as a function of growth rates of demand for i from $t\text{-}2$ to $t\text{-}3$, the growth rate of capacity for $t\text{-}1$ and capacity utilization in $t\text{-}1$, given technology, normal utilization rates and the growth rate of autonomous demand. Thus

$$g_{it}^{m} = g^{m}\left(g_{it-j}^{d}, g_{it-1}^{m}, u_{it-1}^{0}\right)\ \ i = 1,2;\ j = 2,3 \qquad \text{........(II.16)}$$

Completing the model requires modeling the behavior of utilization rates, the ratio of investments of the two sectors (M^0_{12}) and the ratios of autonomous demand to total demand (ID_t^{Au} and CD_t^{Au}). These can be written respectively as

$$u^0_{it} = u_i\left(g^d_{it-1}, g^{de}_{it-k}, g^m_{it-j}\right) \quad i = 1,2;\ k = 1,2;\ j = 0,1 \qquad \text{……..(II.17)}$$

$$M^0_{12t} = \frac{\left(1+g^m_{1t}\right)M^0_{12t-1}}{1+g^m_{2t}} \qquad \text{……..(II.18)}$$

$$CD^{Au}_t = \frac{ID^{Au}_t.\mu}{D_{12t}} \qquad \text{……..(II.19)}$$

where μ (and thus the ratio of autonomous consumption demand to autonomous demand for machines) is given; and

$$ID^{Au}_t = \frac{ID^{Au}_{t-1}.(1+a)}{(1+g^d_{2t})} \qquad \text{……..(II.20)}$$[9]

III Equilibrium

As discussed in White (2008), the model outlined above – when complemented by expressions determining expected rates of growth, g^{de}, for each of the two sectors (Section IV below) – will provide a system of equations describing the time path of ten variables: two expected growth rates of demand, two realized growth rates of demand ($g^d{}_{1t}$, $g^d{}_{2t}$), two growth rates of new capacity ($g^m{}_{1t}$, $g^m{}_{2t}$), two utilization rates on newly installed machines ($u^0{}_{1t}$, $u^0{}_{2t}$), the ratio of induced investments in the two sectors ($M^0{}_{12t}$) and the ratio of autonomous investment demand to total investment demand ($ID^{Aut}{}_t$). With suitable substitutions, the model outlined above can be written as the first-order difference equation system

$$x_t = f\left(x_{t-1}\right) \qquad \text{……..(III.1)}$$

where x is the vector

$$x = \left(g^d_i, g^{de}_i, g^m_i, M^0_{12}, u^0_i, ID^{Aut}\right) \qquad i = 1, 2.$$

An equilibrium (fixed point) of the recursive system (III.1) is characterised by $g^d_{it} = g^d_{it-1} = g^m_{it} = g^m_{it-1} = g*$ and $u_{it} = u^n_i$ for $i = 1,2$.

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[9] Thus it is assumed for simplicity that both autonomous demands grow at a uniform rate.

The discussion that follows is concerned with the dynamic behaviour of system (III.1). That discussion focuses exclusively on the results of a number of computer simulations of system (III.1). Each simulation exercise examines the behaviour of the model over time, starting from a steady state and imposing on the model a shock in the form of a change in the rate of growth of autonomous demands. The values assigned to technology (including normal utilization rates) and the rate of profit, real wage and relative prices for all simulations are given in Tables 1 and 2 in *Appendix II*.

IV Expectations about growth – the basic model without learning

In order to close the model for the purposes of simulation there remains to state explicitly the expected growth rate of demand which underpins decisions about investment (II.8). For the "basic" model on which learning behavior is superimposed, we assume, following White (2008) that, for the purposes of investment at the end of period *t*, producers forecast growth in demand for the period *t+1* based on two calculations: one based on the demand growth in their own sector observed at the conclusion of periods *t-1* and *t-2*; and one based on the growth rates of autonomous demand. The first calculation makes use of the approach of Franke and Weghorst (1988) where producers are assumed to use an average of the previous two realized growth rates, discounted by a factor which depends on the dispersion between those rates.

Thus the estimate of future growth in demand is given by the weighted average

$$g_{it}^{de} = \varepsilon . g_t^{eAut} + (1-\varepsilon) . \frac{\left(g_{it-1}^{d} + g_{it-2}^{d} \right)}{2} . X_{it} \qquad \text{........(IV.1)}$$

where g_{it}^{de} and g_t^{eAut} refer to the expectation held at the end of period *t* respectively about demand growth in sector *i* between the end of periods *t+1* and *t*; and about autonomous demand growth between *t* and *t+1*. X_{it} refers to the abovementioned dispersion factor and is given by

$$X_{it} = \frac{1}{1+\sigma . \left(g_{it-1}^{d} - g_{it-2}^{d} \right)^2} \qquad \text{........(IV.2)}$$

The first term on the right hand side of equation (VI.1) reflects the assumption that persistent growth in the autonomous components of demand leads to the development of expectations about growth in those components of demand. It also reflects the assumption that producers perceive a connection between autonomous demand growth and growth in demand for their own output; viz., that the growth of demand in their own sector is not independent of growth in the economy as a whole and that growth in the aggregate demand is at least partly exogenous.[10] This connection is reflected in the value assigned by producers to the parameter ε.

It has been demonstrated in White (2008) that system (III.1) combined with (IV.1) and (IV.2) allows for steady state growth equal to the rate of growth of autonomous demand.[11] Computer simulations of system III.1 discussed below are based on parameter values (Appendix II) yielding a steady state equal to the rate of growth of autonomous demand (assumed uniform for the two sectors). The ratio of autonomous demand to total demand for each sector is determined endogenously, along with the relative size of the two sectors. In other words, setting all growth rates equal to a, the growth rate of autonomous demands, equations (II.17)-(II.19) would determine ID_t^{Au} and D_{12} and CD_t^{Au}.

For the purposes of simulation it is assumed throughout that the growth rate of autonomous demand is the same for both sectors and that the forecast growth rate of autonomous demand is equal to that recently observed.

Growth rates of demand for this basic model without learning are depicted in Figure 1. The critical parameter here is ε. Panels (a) and (b) indicate that for certain parameter values – including values of σ and ε – the system shows stability in the sense that there exists no tendency for fluctuations to increase in amplitude. For sufficiently high values of ε (> 0.745), the model III.2 – V.1 produces damped cycles in growth rates around the new equilibrium. Thus although the system does not converge to the constant rate of growth of autonomous demand for some ε < 0.745, a sufficient enough weighting of autonomous demand growth in forecasts of growth by producers (i.e. ε sufficiently > 0.2) would render stability at least in terms of a non-explosive fluctuations in growth rates of demand and investment.

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[10] This of course imparts to producers the view that the long-run rate of growth of the economy is not constrained by the available supply of inputs, including labour.

[11] Strictly speaking this analysis also allows for a second type of steady state - certainly more plausible where expectations are not based on expectations about autonomous demand growth along the lines of (IV.1) and (V.2) – where the steady state rate of growth is in effect "endogenous"; viz, where the steady state rate of growth of income exceeds the rate of growth of autonomous demands so that the ratio of autonomous demand to income converges on zero over time.

V Learning and the dynamics of demand

One obvious limitation with the analysis up to this point is that the parameter ε, representing the weighting of elements in the formation of producers' expectations about growth, is exogenous and independent of the economy's behavior over time; and most importantly, independent of errors in producers' growth forecasts.

A first step therefore in improving the model would be to make ε endogenous; that is, as a proxy for making endogenous the views of producers about the relative importance of growth in their own sector and growth in the economy as a whole in their forecasts of growth in demand for their own output.[12]

In this regard, we make use of a procedure suggested by Caminati (1998), viz., the application of a gradient-descent rule applied to the determination of ε for each of the two sectors of the basic model. More precisely it is assumed that ε for each sector is modified in each period on the basis of forecast errors, specifically, how the forecast error itself responded to the change in this weighting the previous period.

The relevant forecast error here is

$$E^{d}_{it+1} = (1/2)\left(g^{d}_{it+1} - g^{de}_{it}\right)^{2} \qquad \text{……..(V.1)}$$

so that the adjustment in the weight, ε_{it}, is given by

$$\varepsilon_{it} = \varepsilon_{it-1} - \eta.\frac{\partial E^{d}_{it}}{\partial \varepsilon_{it-1}} \qquad \text{……..(V.2)}$$

Figures 2 depicts for ,number of different starting values of ε – 0.2 and 0.6 - for each sector, the effect of having ε determined endogenously in accordance with expressions (V.1) and (VI.2); allowing for a number of values of the coefficient η. What stands out immediately is firstly the divergence in ε values for the two sectors even though they were started at the same value for each sector; and secondly, how, the effect of this simple form of learning is to bring about a long-run change in the ε values for both sectors. A third interesting feature (at least tentatively) suggested by these results is that there appears to exist a gravitation around a long-

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[12] Recall that the justification for having views about autonomous demand influencing expectations about growth is as an indicator of the growth rate of the economy as whole.

run “equilibrium” set of ε values: around 0.4. Thus for low starting values – 0.2 – ε for both sectors is driven over time up, while starting values of 0.6 see ε values driven down over time. From V.2, this behavior in the former case implies that increases in ε have the effect at least for a range of ε values of reducing forecast errors; after which ε appears to stabilize; while in the latter case, the opposite holds, so that reductions in ε reduce forecast errors (relatively speaking). The further implication is of course that there is a set of ε values (at least for ε_1, $\varepsilon_2 <$ 0.6), which minimize forecast errors and the simple learning embodied in expressions V.1 and V.2 are able to lead the system to this stabilizing set of ε values. However, by way of qualification to this last point, a fourth feature suggested by figure 2 is that too high a rate of learning (reflected in the value of η) tends to make more volatile the long-run path of ε values. This is suggested by the last two cases depicted in Figure 2, showing starting values of $\varepsilon = 0.6$, but values of $\eta = 3$ and 5 respectively.[13]

One final point in relation to these results is worth remarking on. The tendency for ε values to rise from low starting values is of some importance the regarding the question of how producers might arrive at a value of ε to begin with. As noted in White (2008), “even for the case of long-run convergence of growth rates to that of the rate of growth of autonomous demand – cases with arguably, ‘high’ values of ε [i.e. greater than 0.745] – in general, the growth rates of demand and growth rates of autonomous demand will be unequal at any point in time. Arguably this fact would work against producers assigning a ‘high’ (e.g. sufficient for long-run convergence) value given to ε. Even if producers believe in a long-run gravitation of growth rates around the rate of growth of autonomous demand, uncertainty about the precise dynamics of interaction between growth rates in their own sector and autonomous demands may render this belief of little use for investment decisions”. (pp. 22-23).

The results depicted in Figure 2 suggest one means – via the application of a simple learning rule - by which values of ε evolve to a magnitude consistent with non-exploding fluctuations in growth rates, even though forecast errors are not completely eliminated; and even where initial ε values are such as to give a low weighting to the significance of autonomous demand growth as a determinant of the system’s long-run growth rate.

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[13] This last result appears to be in keeping with results derived by Caminati (**op.cit**. p. 17).

VI Learning II: agent's modeling of the dynamics of demand

We turn now to discussion of what might be considered a more "sophisticated" approach to learning *vis a vis* that of the preceding section. It is now assumed that at least some producers in each sector recognize that realised growth rates for individual sectors, depend, for a given set of growth rates of the economy, past sectoral growth rates, and past growth rates of autonomous demands, on producers' expectations about future growth rates. What is unknown is the nature of the function linking expected growth rates to actual realised growth rates - i.e. not only how past growth rates feed into expectations about future growth, but as well how changes in expected growth rates feedback on realized growth rates.[14] What is unknown in other words is the objective dynamics of the economy.

Thus, any producer wishing to make predictions about growth on the basis of some understanding of the dynamics of demand can be seen to be faced with two tasks: first, working out how other producers generate their expectations about growth on the basis of past growth rates; and, second, working out how these expectations feed into the dynamics of demand.

As to the first task, following the suggestion in Caminati (1989, p. 18), we assume that such an agent – referred to here as 'producer *i*' is able to infer something about the expectations of other producers about sectoral growth rates; If technical coefficients (including especially, desired capital to output ratios) are known, existing capital stocks at the end of *t-1* for example are known and investment expenditures at the end of *t-1* are known, it is possible for producer *i* to form inferences about anticipated sectoral growth rates, the implicit assumption being that producer *i* assumes other producers decide on investment in similar way to himself/herself.

Modeling the second task on the other hand raises more difficult issues. In particular, it requires imparting to producer *i* the capability of or access to modeling of the interaction between investment and aggregate demand, including how autonomous elements of demand fit into that picture. In preceding down this track, the intention here is not to proceed down the "perfect-foresight path" ; viz., which, in the present model, would amount to assuming producer *i* has access to the system (III.1)-(IV.1)-(V.1)-(V.2) above – in other words, access to

[14] Put alternatively, producers are unable to assess how "the actual function mapping the output realizations […. , in our analysis] into current [values, …] changes with every change in the expectation function used to produce the forecast[s, ……] (Caminati, **op.cit**., p. 12)".

(what amounts for the purposes of the present paper to) the "correct" model of the real world. Instead, a more reasonable hypothesis is that producer *i* may have access to a "model" of the present model; e.g. an aggregative one-sector simple-multiplier accelerator model at least for the purposes of explaining how expected growth rates of aggregate demand and growth rates of autonomous demand feed into realized growth rates. As such, that modeling task would involve problems well known to the forecaster, such as assigning values to aggregative versions of sectoral parameters e.g. an aggregate version of the sectoral β's (representing the output capacity of machines) .

With regard to the first of the above-mentioned tasks facing producer *i*, viz., an estimate of the expectations about future growth held by other producers, we assume that estimate, made at the end of period *t*, is equivalent to the determination of g_{it}^{de} in expression (V.1) except that the relevant value of ε is that for period *t-1*.[15] Hence producer *i*'s estimate, g_t^{eij}, formed at the end of period *t* of expected growth rates held by other producers in sector *j* about growth in demand between *t+1* and *t* is given by

$$g_t^{eij} = \varepsilon_{jt-1}.g_t^{eAut} + \left(1-\varepsilon_{jt-1}\right).\frac{\left(g_{it-1}^{d} + g_{it-2}^{d}\right)}{2}.X_{it} \qquad \text{........(VI.1)}$$

The first difficulty which arises however is that producer *i* seeks a simplified "workable" representation of expectations about growth held by other producers, but the ("real world") model of sections II – V above is multi-sectoral. Hence producer *i* is faced with the task of reducing different expectations about demand growth in each the two sectors into an "aggregate" expectation about growth. To this end it is assumed that producer *i* uses a weighted average of the g_t^{eij} estimates for the two sectors where the weighting is determined by the most recent observation (end of period *t-1*) of the relative size of demands for the two sectors.

[15] Since, if producer *i* supposes in accordance with V.1 and V.2, that ε in each sector is determined endogenously, ε_t could only be determined on the basis of demand in each sector at the end of *t* being known. But such demand cannot be known prior to producer *i* calculating his/her own investment demand. Effectively, expression (VI.1) implies that producer *i* adjusts the value of ε each period in a simple adaptive expectations manner. It is further assumed here that producer *i* assumes that other producers form their expectations, in the absence of modeling, in a manner similar to what he/she would have otherwise done, including values for σ and η (expressions IV.2 and V.2 respectively) which he/she would have otherwise used.

Thus producer i's estimate of the "aggregate" growth expectation held by other producers at the end of period t (about growth in demand between the end of periods t and $t+1$) is given by

$$g_t^{ei} = \left(\frac{\kappa.Q_t}{1+\kappa.Q_t} \right).g_t^{ei1} + \left(1 - \frac{\kappa.Q_t}{1+\kappa.Q_t} \right).g_t^{ei2} \qquad \text{........(VI.2)}$$

where

$$Q_t = M_{12t}^{0}.\frac{\beta_1}{\beta_2}.\frac{u_{1t}^{0}}{u_{2t}^{0}}$$

and is taken as a proxy for the relative size of demands for the two commodities during period t. Having estimated other producers' collective views about future growth at the end of period t, producer i turns to the second of the above-mentioned tasks, viz., working out how this expectation feeds into the actual investment decisions and thus into actual growth, specifically, between the end of periods t and $t+1$. And on this basis, producer i will, it is assumed, make his/her own decisions about production and investment,

To this end and following the suggestion above, it is supposed that producer i constructs a simplified aggregate version of the model (III.1)-(IV.1)-(V.1)-(V.2) which can serve as the basis of producer i's prediction of future demand growth in the economy. The simplest form of such a model could be written as:

$$\begin{aligned} D_t &= C_t + I_t + D_t^{Aut} \\ C_t &= c.Y_{t-1} \end{aligned} \qquad \text{........(VI.3)}$$

In view of expression II.6, the consumption function can be written as

$$C_t = c.\frac{D_{t-1}\left(1+g_{t-2}^{de}\right)}{1+g_{it-1}^{d}} \qquad \text{........(VI.4)}$$

The modeling of investment follows broadly the approach adopted in the model (III.1)-(IV.1)-(V.1)-(V.2) though we assume that producer i's modeling of this is somewhat simpler, in particular by collapsing the analysis of two sectors down to an analysis of one sector. Hence, a one-sector version of expressions II.6 – II.9 yields the corresponding one-sector version of II.10,

$$I_t = \frac{1}{\beta}\left\{D_{t-1}\cdot\left(\frac{\left(1+g_t^{de}\right)^2}{u^n} - \frac{\left(1+g_{t-1}^{de}\right)\left(1+g_t^m\right)}{u_t^0\left(2+g_t^m+\phi+g_t^m\phi\right)}\right)\right\} \qquad \text{……..(VI.5)}$$

where g_t^m represents (as earlier, though for the one-sector case) the growth rate of the capital stock (machines) between periods t and t-1. It is assumed that producer i takes the rate of growth of demand for commodity 2, g_{2t-1}^d, which can be observed between the end of periods t-1 and t-2 as a proxy for the growth rate of the capital stock.

Solving expression (VI.5) backward for I_{t-1}, and combining with (VI.3)-(VI.4) allow one to derive an expression for the growth rate of aggregate demand, g_t^{mod}, between the end of periods t and t-1; as a function of past growth rates in demand, the capital stock, expected demand as well as expected demand growth at the end of period t and the ratio of autonomous demand D_t^{Aut} to total demand D_t. This function represents in effect producer i's prediction of the growth of demand which would take place between the end of periods t and t-1 on the basis of expectations about future growth; this latter being estimated in turn by producer i on the basis of expression (VII.2) above. Thus

$$g_t^{mod} = g^{mod}\left(g_{t-1}^{mod}, g_{t-k}^{ei}, g_t^m, u_t^{mod\,0}, D_{mod\,t}^{AD}\right) \qquad k = 0,\, 2 \text{ and } 3 \qquad \text{……..(VI.6)}$$

where $u_t^{mod\,0}$ and $D_{mod\,t}^{AD}$ refer respectively to producer i's modeled values of aggregate utilization during period t and the ratio of autonomous demand to total demand at the end of period t. With regard to $u_t^{mod\,0}$, producer i's modeling of utilization for period t can be based on observables when the estimate is calculated and it is assumed that this estimate will be a weighted average of the utilization rates across the two sectors.

Modeling of $D_{mod\,t}^{AD}$ on the other hand is slightly more complex. Part of the complexity lies in the fact that at the end of period t when producer i has to model g_t^{mod}, $D_{mod\,t}^{AD}$ cannot be observed, since D_t and thus I_t^{Aud} cannot be observed . There is also the realization by producer i that this ratio is subject to considerable fluctuation from period to period. In view of these two considerations, it is assumed that take an average of past D_{mod}^{AD}'s to calculate $D_{mod\,t}^{AD}$. For this purpose it is assumed that producer i has access to national accounts information about the economy, sufficient to enable a calculation of $D_{mod\,t}^{AD}$ for any period t in the past, at least in nominal terms. More precisely, the ratio of autonomous demand measured in nominal terms as a proportion of nominal aggregate demand which would be implied by the model (III.1)-(IV.1)-(V.1)-(V.2)

$$D^{AD}_{mod\,t} = \frac{ID^{Aut}_{t-1}.\left(p_{21}.u^{n0}_{2}.\beta_2 + M^{0}_{12t}.u^{n0}_{2}.\beta_1.\mu\right)}{\left(M^{0}_{12t}.u^{n0}_{1}.\beta_1 + p_{21}.u^{n0}_{2}.\beta_2\right)} \qquad \text{........(VI.7)}$$

Hence, it is assumed that producer *i* calculates g^{mod}_{t} as

$$g^{mod}_{t} = g^{mod}\left(g^{mod}_{t-1}, g^{ei}_{t-k}, g^{m}_{t}, u^{mod\,0}_{t}, D^{AD}_{Histt-1}\right) \qquad \text{........(VI.8)}$$

where

$$D^{AD}_{Hist} = \sum_{i=1}^{n}\chi_i.D^{AD}_{mod\,t-i} \qquad \text{........(VI.9)}$$

An analogous approach is adopted for the consumption propensity, '*c*' in expression (VI.3), so that

$$c = c^{Hist}_{t} = \sum_{i=1}^{n}\psi_i.c^{mod}_{t-i} \qquad \text{........(VI.10)}$$

where

$$c^{mod}_{t} = \frac{\left(1+g^{d}_{1t}\right).\left(1+g^{d}_{1t-1}\right).M^{0}_{12t-2}.u^{n0}_{1}.\beta_1}{\left(1+g^{de}_{1t-2}\right).M^{0}_{12t-2}.u^{n0}_{1}.\beta_1 + \left(1+g^{de}_{1t-2}\right).p_{21}.u^{n0}_{2}.\beta_2} \qquad \text{........(VI.11)}$$

Similarly to $D^{AD}_{mod\,t}$ of expression (VI.7) above, the RHS of expression above is what the model (III.1)-(IV.1)-(V.1)-(V.2) would generate as the value of consumption demand as a share of aggregate demand, c^{mod}_{t}, and thus what producer *i* could glean each period from a set of national accounts.

Of course the task for producer *i* who is planning investment and output like any other producer is really to forecast growth in demand between the end of periods *t+1* and *t*, so that the modeling task for producer *i* therefore involves solving expression (VI.6) forward by one period and thus solving

$$g^{mod}_{t+1} = g^{mod}\left(g^{mod}_{t}, g^{ei}_{t-k}, g^{m}_{t+1}, u^{mod\,0}_{t+1}, D^{AD}_{Histt}\right) \qquad k = -1,\ 1 \text{ and } 2 \qquad \text{........(VI.12)}$$

As indicated by expression (VI.12), producer *i*'s forecast of growth in demand between the end of periods *t+1* and *t* will be based in part on producer *i*'s forecast of growth in demand between the end of periods *t* and *t-1*. Indeed, the arguments g^{m}_{t+1} and $u^{mod\,0}_{t+1}$ on the RHS of expression (VI.12) will at the end of period *t* also be "forecasts".

With regard to the forecast $u_{t+1}^{mod\,0}$ an aggregative, one-sector version of (II.16) is

$$u_t^0 = \frac{Y_t.(1+\phi)}{\beta.(M_t^0.(1+\phi)+M_{t-1}^0)} = \frac{D_{t-1}.(1+g_{t-1}^{de}).(1+\phi)}{\beta.(I_{t-1}^0.(1+\phi)+M_{t-1}^0)} \qquad \text{.......(VI.13),}$$

so that, in view of expression (VI.5), and solving forward for period t+1, one can arrive at

$$u_{t+1}^{mod\,0} = u^0\left(g_{t-h}^{mod}, g_{t-k}^{ei}, u_{t-n}^{mod\,0}, g_{t-j}^{m}\right) \qquad h = 0,1;\ k = 0,\ 1,\ 2;\ n = 0,\ 1;\ j = 0,\ 1 \qquad \text{........(VI.14)}$$

With regard to the forecast g_{t+1}^{m} in (VI.12) it is assumed for simplicity that producer *i* takes as an indicator of the likely growth of capacity between periods *t+1* and *t* his/her estimate of the expectation of demand growth between the end of these two periods, viz., g_{t+1}^{ei}. In turn, it is further assumed that producer *i* determines g_{t+1}^{ei} along the same lines as g_t^{ei} in (VI.2) above, so that

$$g_{t+1}^{ei} = \left(\frac{\kappa.Q_t}{1+\kappa.Q_t}\right).g_{t+1}^{ei1} + \left(1-\frac{\kappa.Q_t}{1+\kappa.Q_t}\right).g_{t+1}^{ei2} \qquad \text{........(VI.15)}$$

where

$$g_{t+1}^{eij} = \varepsilon_{t-1}.g_t^{eAut} + (1-\varepsilon_{t-1}).\frac{\left(g_t^{mod}+g_{jt-1}^{d}\right)}{2}.X_{jt+1} \qquad j = 1,2 \qquad \text{........(VI.16)}$$

with

$$X_{jt+1} = \frac{1}{1+\sigma.\left(g_t^{mod}-g_{jt-1}^{d}\right)^2} \qquad \text{........(VI.17)}$$

In other words, it is supposed that producer *i* estimates g_{t+1}^{ei} by assuming that other producers form their expectations along the lines of (IV.1), (IV.2) and (V.1), (V.2) (as for the estimation of g_t^{ei}), but the value of the growth rate g_{jt}^{d} required for that calculation (for each of the two sectors, *j* = 1,2) is the rate g_t^{mod} modeled by producer *i* at the end of period *t*. The weighting of the two sectors in the calculation of g_{t+1}^{ei} will be analogous to that in the calculation in VI.2 above, viz., on the basis of the relative size of the two sectors.

In summary, expressions (VI.1) – (VI.17) provide a means by which producer *i* in any sector could model the growth rate of aggregate demand between periods *t* and *t+1*. The final question concerns the weighting that producer *i* places on this modeled growth rate in formulating his/her own investment decision at the end of period *t* and his/her own decision about output to be produced in period *t+1*. It is assumed that producer *i* in any sector *j*, calculates the expected growth rate of demand in his/her own sector between periods *t+1* and

t as a weighted average of the modeled growth rate of the economy - g_{t+1}^{mod} (expression (VI.12)) - and the growth rate, g_{jt}^{epst} which he/she would have calculated on the basis of past growth rates of demand, in the absence of any modeling (i.e. (IV.1), (IV.2), (V.1) and (V.2)), but without consideration of autonomous demands. Hence, g_{jt}^{epst} is equivalent to the calculation of IV.1 and IV.2 but with ε = 0. Thus producer *i*'s calculated growth rate for his/her own sector *j* between periods *t+1* and *t* is therefore

$$g_{jt}^{dei} = \omega . g_{t+1}^{mod} + (1-\omega) . g_{jt}^{epst} \qquad \text{........(VI.18)}$$

where

$$g_{jt}^{epst} = \frac{\left(g_{it-1}^{d} + g_{it-2}^{d}\right)}{2} . \frac{1}{1+\sigma.\left(g_{jt-1}^{d} - g_{jt-2}^{d}\right)^{2}} \qquad \text{........(VI.19)}$$[16]

Additionally, we suppose that only (a proportion ρ of) some producers in any sector formulate expectations along the lines of producer *i* so that for each sector *j* as a whole, the expectation of growth in demand held at the end of period *t* will be given by

$$g_{jt}^{de} = \rho . g_{jt}^{dei} + (1-\rho) . g_{jt}^{ewl} \qquad \text{........(VI.20)}$$

where g_{jt}^{ewl} is the growth rate calculated on the basis of simple learning, i.e. on the basis of expressions (IV.1), (IV.2), (V.1) and (V.2).

VII "True learning" I: exogenous ρ and ω

We briefly discuss here the results of simulations of the model with "true learning", with the magnitude of the parameters ρ and ω set exogenously; a positive value of the coefficient ρ (equation (VI.20)), representing the magnitude of "true" learning within the aggregate formation of expectations in each sector. These results are depicted in Figure 3: specifically for two cases with starting values of ε for each sector set at 0.2 and 0.6 respectively and with η = 1.5. For each simulation case, Figure 3 shows growth rates of demand, values of ε for the

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[16] ε is zero in the calculation of g_{jt}^{epst} because the influence of autonomous demands on growth of the economy and growth in producer *i*'s own sector is already factored into the model used by producer *i* and thus into the forecast, g_{t+1}^{mod}.

two sectors, utilization rates for the two sectors as well as the evolution of the ratio of autonomous demand for machines as a proportion of total investment demand, ID_t^{Aut}.[17]

One interesting feature of some of the simulation with true learning compared with the simple learning case is that the true learning leads to a more pronounced increase in epsilon values for both sectors, over the long-run. The significance of this fact is that the use of a modeled forecast by a sufficient proportion of producers in each sector (a sufficiently high enough ρ), and with enough weight given to the model's forecasts by that group of producers (a sufficiently high enough ω) impacts positively on what those engaged in simple learning see as the accuracy of their forecasts.[18] In other words, the latter group of producers are finding that their forecast errors are declining when they increase their epsilon – i.e. the weighting they give to autonomous demands in their calculation of expected future growth rates of demand in their own sector. Not surprisingly, as is evident from Figure 3, epsilon in some cases is pushed up sufficiently far to generate a clear convergence of sectoral growth rates towards the uniform rate of growth of autonomous demands.

An interesting implication of this first set of results is that modeling of the economy as a means of anchoring expectations to some understanding of the dynamics of demand can significantly impact on those dynamics, even though it is only some producers who formulate their expectations this way. In other words, the "true" learning of some producers may obviously be sufficient to impact significantly on the expectations of all producers in a manner which could add to the stability of the system's dynamics.

VIII "True learning" II: endogenous ρ and ω

These first simulations with "true learning" raise the obvious question as to what forces govern the magnitude of ρ and ω? Those simulations certainly suggest different results arising from differences in the values of these two parameters. But how do those parameters evolve through time. The equally obvious first answer would be that they evolve based on the relative

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[17] Recall, as equation (II.19) indicates, this ratio can be linked to the corresponding ratio relating to demand for the consumption good.

[18] This must be the case since for those producers using a model-based forecast the non-model part of their forecast is based simply on past realised growth rates in their own sector and not on growth rates of autonomous demand (as noted above, the latter are allowed for in the modelled component of the true learning forecast).

success of the two different types of learning – simple versus true learning. In more conventional terms, as Franke and Sethi argue, "differential payoffs exert evolutionary pressure. In this case … in general optimizers as well as non-optimisers are admitted but in each period the group performing better in the recent past wins some converts. The basic question then becomes: will one group (the non-optimisers?) be completely competed away, or is there scope for co-existence even in the long-run?" (1995, p. 584).

The interpretation of this sentiment given here is that the proportion of producers making use of a modeled forecast and thus engaging in "true" learning is a function of the relative accuracy of the forecasts of the growth of the economy compared with the forecasts generated by a the simple learning procedure of section VI above. For the purposes of the present analysis the "payoff" of each "strategy" is thus assumed to basically reflect the accuracy of the forecast.[19]

Hence we assume that ρ would be smaller where the forecast error associated with the producer's prediction of their sectoral growth rate given by the simple learning procedure is smaller than the forecast error in the modelled growth rate with respect to the aggregate economic growth rate. More precisely, for the purposes of simulation we assume that

$$\rho_{jt} = \rho_{jt-1} \cdot \left(1 + \frac{\left(\left(g_{jt-1} - g_{jt-2}^{ewl} \right)^2 - \left(g_{t-1} - g_{t-1}^{mod} \right)^2 \right)}{1 + \left| \left(\left(g_{jt-1} - g_{jt-2}^{ewl} \right)^2 - \left(g_{t-1} - g_{t-1}^{mod} \right)^2 \right) \right|} \right) \quad \text{……..(VIII.1)}$$[20]

Thus, for example, if the forecast associated with simple learning exceeds that associated with the prediction of the model utilized by producer *i*, the numerator of the parenthetical term on the right-hand side of (VIII.1) will be positive and the parantheical term itself positive and greater than unity, so that forecasting on the basis of a model "wins some converts", viz., ρ rises. Conversely, if simple learning yields a smaller forecast error, the numerator of the

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[19] Indeed, the "payoffs" for the individual producer must ultimately reflect the extent to which that individual producer is able to capture a greater share of the profit flow in his/her market. This cannot but be related to the extent to which the producer's forecasts of demand are more accurate than their competitors.

[20] The more difficult issue is the amount of information which producers can obtain, for a feasible cost, in relation to modeling and its forecasts. For this purpose one might suppose that the modeled growth rates are the construct of various institutions, e.g. large private sector banks, policy-making bodies such as central banks and treasuries etc. In turn, in a fuller analysis, this cost would need to be factored into the calculation of "payoffs" associated with engaging in "true" learning. Needless to say, these costs are ignored in the present analysis.

paranethetical term is negative and the parenthetical term itself will be positive but less than unity so that ρ falls.

As to the endogenous determination of ω, a analogous approach is adopted: it is assumed that ω - reflecting the importance that those engaged in "true learning" give to the model forecast as against previous growth rates in their own sector in working out their overall growth forecast – is based on the relative accuracy of those two components. In other words, the determination of ω is similar to IX.1 above but with g_{jt}^{epst} (expression VI.19) replacing g_{jt}^{ewl} so that

$$\omega_{jt} = \omega_{jt-1} \cdot \left(1 + \frac{\left(\left(g_{jt-1} - g_{jt-2}^{epst} \right)^2 - \left(g_{t-1} - g_{t-1}^{mod} \right)^2 \right)}{1 + \left| \left(\left(g_{jt-1} - g_{jt-2}^{epst} \right)^2 - \left(g_{t-1} - g_{t-1}^{mod} \right)^2 \right) \right|} \right) \qquad \text{……..(VIII.2)}$$

With (VIII.1) and (VIII.2) the simulations of the previous section were re-run, using the values of ρ and ω for those simulations as initial values for the re-run simulations.

Results for these simulations shown in Figure 4. As is evident the possibility exists of long-run decline in both ρ and ω as well as a long-run increase and also of ρ and ω moving in opposite directions over the long-run.

Specifically there are three particularly interesting aspects to the results. First, an endogenous ρ and ω does not necessarily stabilize the system – in terms of reducing the size of the fluctuations in growth rates of demand and in cases where it appears to increase the size of these fluctuations (e.g. compare the first and third cases in Figure 3 with those of Figure 4), this is associated with ε for both sectors being pulled down instead up (as in the cases with exogenous ρ and ω). In these cases, both ρ and ω in both sectors are declining throughout the simulation. In other words, not only are some producers shifting away from a modeled forecast towards one based exclusively on simple learning – because the latter is more accurate; but as well, those engaged in "true" learning are giving less weight over time to the model in formulating their forecasts about the growth of demand – because forecasts of sectoral growth based on past growth rates are more accurate.

A second interesting feature of the results is that for the same starting values of ρ and ω, the long-run dynamics are clearly sensitive to the initial values of ε for each sector (compare for example panels (a) and (d), (b) and (e), (c) and (f) in Figure 4); and intriguingly show greater convergence of growth rates to those of autonomous demand where the starting value of ε is lower, rather than higher. The likely explanation for this may well be found in the explanation for a third interesting feature of the results – the apparent long-run co-existence of the two different approaches to learning.

This "co-existence" of the two groups is represented by ρ showing no long-run tendency to converge either on 1 (all producers making use of a modeled forecast) or zero (all producers engaged in "simple learning"). What is obviously behind the possibility of co-existence is that over time, no one approach "wins" out for all producers. The interesting question is why? The answer not-surprisingly lies in the fact that changes in the proportions of producers engaged in the two types of learning will impact on the errors for both groups. Some of the simulation results suggest that as more producers engage in true learning, to the extent that this brings the realized growth rates into closer alignment with the rate of growth of autonomous demand, it would also impact positively on the parameter ε. In other words, producers engaged in simple learning would find that if they increase ε, this corresponds with a reduced error in their growth forecasts – although, this reflects as much the greater proportion of producers engaged in true learning and the effect of that in bringing growth rates closer to that of autonomous demand. If ε rises sufficiently the error from simple learning will fall below that associated with the model and some producers would move back towards a simple learning behavior in preference to true learning. In this sense, there may be a long-run *positive* feedback of an increasing proportion of those engaged in true learning back on the relative performance of the simple learning behavior: this in turn, limiting or even reversing the shift towards true learning. Further, this might suggest that any stabilizing impact of "true" learning may be more pronounced where the system has less of a history of simple learning and thus "begins" with lower ε's.

IX Concluding notes

The analysis above represents the first steps of an inquiry into the significance of learning in a demand-led growth model. Though preliminary in nature, the results canvassed above are sufficient to indicate some impact of learning either in terms of a simple gradient-descent rule or a more sophisticated modeling procedure on the part of some producers on the dynamics of demand; and interestingly on the relation between growth rates of aggregate demand and growth in autonomous components of demand.

The results suggest further that this impact need not always act to reduce fluctuations in growth rates and that, compared with the absence of learning, any stabilizing impact of learning depends in part on the speed with which producers learn and the prevailing conditions when learning begins (i.e. the initial conditions).

Clearly, further research should aim at extending the analysis in three respects at least. The first is to allow for fluctuations in the rates of growth of autonomous demand and therefore investigating the extent to which the results presented here are conditioned by the assumed constant and uniform rates of growth of autonomous demand.

A second extension to the analysis would allow for changes in relative prices: this is particularly pertinent for the investigation of models with multiple production processes. The classical-Sraffian literature on price gravitation (Caminati and Petri, 1990) offers considerable insight into how the model may be extended in this regard. With relative prices influenced by the movement of capacity (supply) relative to demand in different degrees across different industries/sectors, the initial shock in the growth rates of autonomous demand would conceivably open up differentials in profit rates across those sectors. In turn investment in new capacity would respond not only to anticipated growth in demand but also to relative profit rates. In fact, the extent of the former response may be made to depend on relative profit rates. The learning process could then conceivably be extended to learning about the significance of differentials in sectoral profit rates.

A third and final extension to the analysis would allow for changes to aspects of the model used by producers engaged in "true" learning, rather than simply having the relative accuracy of the model's forecasts feedback only on the significance attached to the model's forecasts by those producers.

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Appendix I:

As indicated in the Introduction, the model takes relative prices and the real wage as given at their long-period equilibrium levels, consistent with a uniform rate of profit and determined along the lines of Sraffa (1960) for the case of joint production (*cf.* White, 2008, p.11).

More precisely, relative prices are the solutions to the following price system:

$$\begin{aligned} M^{0}_{it}.p_{21t}.(1+\pi_t)+w_t.l^{0}_{i}.Y^{0}_{it} &= Y^{0}_{it}.p_{i1}+M^{0}_{it}.p^{m11}_{it} \\ M^{1}_{it}.p^{m11}_{it}.(1+\pi_t)+w_t.l^{1}_{i}.Y^{1}_{it} &= Y^{1}_{it}.p_{i1} \end{aligned} \qquad i = 1,2 \quad .....(A1.1)$$

where Y^0_{it} and Y^1_{it} refer to outputs of commodity *i* on new and old machines respectively. The value of the used machine is equal to the profit per unit of output on the machine, discounted at the rate of profit, π, assumed to be uniform across the two production processes. It is also assumed here that equilibrium is maintained in the market for used machines.

Assuming that relative prices prices correspond to the normal (desired) utilization of productive capacity, price equations (A1.1) can be rewritten as

$$p_{21}.(1+\pi)=u_i^n.\beta_i.\left(p_{i1}-w.l_i^0\right)+p_i^{m11}$$
$$p_i^{m11}.(1+\pi)=u_i^n.\beta_i.\left(p_{i1}-w.l_i^0.(1+\alpha)\right)\Big/(1+\phi) \qquad i=1,2 \quad \text{.....(A1.2)}$$

where u^n refers to the desired / normal utilization rate on newly installed capacity, which with ϕ also given implies a "desired" utilization rate on older machines. The price system (A1.2) determines three relative prices and the real wage rate for an exogenously determined rate of profit.

Appendix II: Simulation details – values for parameters and lagged endogenous variables

Table 1

l^0_1	= 0.1	p^m_{11}	= 1.44322	w	= 0.26180
l^0_2	= 2	p^m_{12}	= 1.42869	α	= 0.1
β_1	= 2	p_{21}	= 2.9795	μ	= 0.2
$\beta 2$	= 0.8	π	= 0.04		

Table 2

g^d_{it-j} $j=1…4$	0.04	ID^{Aut}_{t-1}	0.159
g^m_{it-1}	0.04	M^0_{12t-1}	0.206
u^0_{it-1}	0.85		

As noted above, simulations of the model involve starting the model in a steady state (corresponding to a rate of growth of autonomous demand of 4%) and then subjecting it to a shock in the form of a permanent increase in the rate of growth of autonomous demand (from 4 to 5 %). Apart from this, calibration was only required for the coefficient σ (see the discussion of this in White, 2008 p. 17).