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On the Role of Profits-Wages Ratios in the Determination of the Long-Run Behavior of International Relative Prices

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Abstract

This paper presents a reconstruction and evaluation of the theory of international relative prices (IRP) based on the theory of 'real competition'. The main thesis of the theory is that the long-run behavior of the IRP of tradable commodity bundles is exclusively determined by their relative total unit labor costs (RTULC). This is equivalent to the assertion that the total profits-wages ratios (TPWR) of these two bundles are sufficiently similar over time and, therefore, neutral in the long run. We identified a set of issues that cast doubt on the strength of the theory. Firstly, due to accounting considerations, the proposed hypotheses cannot constrain IRP to depend solely on the RTULC. Secondly, the theoretical and empirical arguments put forth by the literature to constrain the TPWR are weak. The paper presents a comprehensive investigation of industries' TPWR, which reveals that their statistical regularities do not align with the constraints necessary for the validity of the theory's core thesis.

Keywords: real exchange rates; terms of trade; absolute advantage; unit labor costs; capital intensities.
JEL Class: B51; D57; D33.

1 Introduction

The relationship between the international relative prices (IRP) of commodities and their relative unit labor costs is relevant to international economics, particularly to the study of the competitiveness and trade of nations. The IRP can be the terms of trade between two commodity bundles or the bilateral or effective real exchange rates of a nation. Some authors maintain that the long-run behavior of IRP *depends* on its associated relative unit labor costs. Others *define* the equilibrium IRP as these relative costs. These two views have originated from both neoclassical and critical traditions in economic theory. Whereas the former is based on the purchasing power parity (PPP) hypothesis and the principle of comparative advantage,[1] the latter originate from the principle of absolute advantages.[2]

One of the most frequently applied theories of IRP based on the principle of absolute advantages is the one developed by Shaikh (1980; 1991; 1995; 1999; 2016, ch. 11). This theory combines Shaikh's work on 'real competition' (Shaikh, 2016, ch. 7) and on the labor theory of value and industrial prices (Shaikh, 1984; 1998; 2012; 2016, ch. 9).[3] The main thesis of Shaikh's theory is that the long-run behavior, or center of gravity, of the IRP of any pair of tradable commodity bundles is determined by their relative *total* unit labor costs (RTULC).[4] It is argued that while forces of competition are sufficiently strong to generate relative market prices to gravitate around relative production prices, i.e., prices that generate uniform profit rates, relative production prices have as backbone the RTULC. As will be demonstrated in Section 2, the relative prices of any pair of tradable commodity bundles, whether produced domestically or internationally, are equal to their RTULC if and only if their ratios of total profits to total wages are equal. Therefore, the thesis of the long-run determination of IRP exclusively by its RTULC is equivalent to the thesis of the *long-run neutrality* of the relative total profits-wages ratios (RTPWR). To

[1]Krueger (1983, ch. 2) and Isard (1995, ch. 4 & 6) provides a thorough exposition of IRP based on the neoclassical tradition. Officer (1976, p. 10-3), Marsh and Tokarick (1994), and Hinkle and Nsengiyumva (1999) provide a survey where, based on cost parity approach to PPP, unit labor costs are used as a measure of competitiveness and of the degree of misalignment of the real exchange rate.

[2]See, inter alia, Shaikh (1979, 1980), Dosi et al. (1990), and Bellino and Fratini (2022).

[3] Other approaches within the principle of absolute advantages use linear production models to extend the Sraffian model of production prices to international trade –see, e.g., Parrinello (2009), Vasudevan (2012), and Bellino and Fratini (2022) and the literature cited there. Other studies base their approach on Pasinetti's (1981) structural dynamics approach, like in Garbellini (2021) and Machado and Trigg (2021). All these approaches consider prices to be the most important element in firms' or industries' competitiveness. However, there are other approaches within the principle of absolute advantages, like the literature on technology-gap (see, for instance, Milberg 1994), which consider that firms' competitive position depends on other variables in addition to prices. Aglietta and Oudiz (1984) define the real exchange rate with the relative unit labor costs and build a model to explain the long-run behavior of the latter.

[4] 'Total' refers to the direct and indirect labor costs, that is, the vertically integrated labor cost.

obtain this result, the literature proposes theoretical and empirical arguments that make the RTPWR of industries *within countries* a disturbance term centered around one, with the expectation that this will constrain the RTPWR of the *internationally* produced commodities.

The literature based on Shaikh's theory presents empirical evidence in support of the hypothesis of a long-run association between IRP and RTULC for developed and underdeveloped economies and for different indicators of IRP, such as bilateral real exchange rates,[5] the real effective exchange rates,[6] and the terms of trade between pairs of commodities.[7] The regressions typically include control variables (such as government expenditure and GDP), other variables capturing short-term determinants of IRP (such as capital flows and interest and profit rate differentials), and a factor controlling for the non-tradable commodities included in the data. Due to the limitations of available data, the evidence presented in some cases concerns the direct, rather than the total, unit labor costs. This evidence suggests that the long-run association between IRP and their relative unit labor costs can be regarded as a stylized fact of modern economies. But how solid are the theoretical and empirical foundations that supports the theory?

In our reconstruction and evaluation of Shaikh's theory, we have identified numerous issues with its fundamental premises. The primary issue is that the key hypothesis *is not efficient* substantiating the proposed thesis. The hypothesis posits that in the long run the relative prices between any two commodities *within a country* equal their RTULC. This is equivalent to the constraint of long-run homogeneity in total profits-wages ratios *within a country.* Therefore, one first problem of the thesis is that it requires *international, and not domestic,* homogeneity of total profits-wages ratios. This necessary (as opposed to proposed) hypothesis contradicts the criticisms of this literature regarding the neoclassical assumptions of homogeneity in the techno-distributive conditions of nations. These assumptions are described as unrealistic. We will demonstrate that even if we assume uniformity in total profits-wages ratios within countries, *IRP are still affected by the relative wage shares in the long-run.*

Secondly, the arguments put forth by the literature to justify their proposed hypotheses are *weak*. The *theoretical* arguments, such as the existence of strong inter-industry connections, are, at this stage, merely speculations, as they do not yet prove that they can constrain the national nor international RTPWR to be

[5] See Boundi-Chraki (2017), Martínez-Hernández (2010), Moreno-Rivas (2018), Ruiz-Nápoles (2001, 2010, 2023), Shaikh (2002), and Shaikh and Antonopoulos (2012).

[6] See Antonopoulos (1999), Ersoy (2010), Góchez-Sevilla and Tablas (2013), Kvangraven (2018, ch. 2), Martínez-Hernández (2017), Papafragkos (2023), Poulakis and Tsaliki (2023a,b), Ruiz-Nápoles (2010), and Stravelakis (2022).

[7] See Boundi-Chraki (2019, 2020) and Boundi-Chraki and Perrotini-Hernández (2021).

neutral in the long run. Regarding the *empirical* arguments, the situation is even more problematic. There is a dearth of knowledge regarding the empirical properties of industries' profits-wages ratios in the literature. Consequently, not only is there a lack of empirical evidence to support the hypotheses, but the empirical arguments used in the literature are also unsubstantiated.

This paper addresses the aforementioned gap in the literature by conducting the first large-scale study of industries' profits-wages ratios. To this end, the WIOD database, comprising data from 42 countries over the period 2000-2014 for up to 56 industries, was utilized. The results show that the statistical patterns observed in the total profits-wages ratios challenge the constraints posited by Shaikh's thesis. The ratios exhibit a striking degree of persistence in their empirical distribution, displaying a statistical propensity to cluster around central values with limited variability and asymmetry. However, the variability of the central tendencies of these ratios *across countries* make it challenging to maintain that the total profits-wages ratios of any two industries will be sufficiently similar for the IRP to be determined in the long-run *exclusively* by its RTULC. Therefore, *there is evidence that the total profits-wages ratios are not neutral in the long-run determination of IRP.*

Shaikh and his followers posit that their theory is based on the classical or 'the real' theory of competition. Nevertheless, none of the characteristics inherent to these theories can be employed to render the RTPWR of any two pairs of domestic or international commodity baskets neutral in the long-run determination of relative prices. In the most favorable scenario, they can establish global uniformity in wage and profit rates, yet *capital intensities may remain disparate.*

The remaining sections of the paper are as follows: Section 2 establishes an economic framework wherein we can (i) examine the *accounting structure* of national and international relative prices at market and production prices and (ii) represent Shaikh's theory of IRP. Section 3 introduces the theory of 'real competition' and the hypotheses and arguments proposed to derive the theory's main thesis. It then evaluates the effectiveness of the hypotheses in producing the theory's results, as well as the theoretical and empirical foundations of these hypotheses. Section 4 studies the statistical properties of industries' profits-wages ratios and reports a lack of evidence for the necessary hypotheses. Section 5 concludes by developing some implications for related literature.

2 National and International Relative Prices: Accounting Structure and Production Prices

2.1 A general model of capitalistic market economy

Suppose that the world economy is composed of two countries, A and B, each one with its own currency, fulfilling the following assumptions. The two countries are capitalist economies with no government and producing n divisible commodities by single-product industries. Land is abundant and labor is indispensable, thus value-added is positive in each industry. Let $N = \{1, 2, \ldots, n\}$ be the set of all labels of commodities produced. The price of commodity $j \in N$ is uniform across buyers and sellers —the law of one price holds. *All commodities are tradable, however, there is no trade in commodities used as means of production.*[8] No assumptions are made regarding the input-output relations, the nature of the labor inputs, the composition of the capital advanced, and the rates of return to workers and capital, except for the homogeneity of conditions of production and reproduction of capitals within industries. All variables are functions of time.

2.2 National relative prices (NRP) at market values

The cost-of-production decomposition of the output value of industry $j \in N$ is

$$x_j = \text{wages}_j + \text{profits}_j + \text{value of means of production}_j. \tag{1}$$

Given that $x_j = p_j q_j$, where p_j is the market price of the j-th commodity and q_j its quantity produced, p_j can be decomposed as

$$p_j = \omega_j + \pi_j + \lambda_j \quad \text{for} \quad j \in N, \tag{2}$$

where $\omega_j \equiv \frac{\text{wages}_j}{q_j} > 0$, $\pi_j \equiv \frac{\text{profits}_j}{q_j} \geqslant 0$, and $\lambda_j \equiv \frac{\text{means of production}_j}{q_j} \geqslant 0$ represent the labor costs, the profits, and the means of production costs in industry j, all of them per unit of output j. Because ω_j and π_j represent the unitary wages and profits for industry j, we will refer to $\psi_j \equiv \frac{\pi_j}{\omega_j} \geqslant 0$ for $j \in N$ as the *direct* profits-wages ratio.

From the Smithian value decomposition[9] we can conduct the vertical integration of the value-added in λ_j, that is, aggregate the wages and profits in the value chain of one unit of commodity j, and obtain $\lambda_j = \omega_j^I + \pi_j^I$, where ω_j^I and π_j^I are the

[8] It is acknowledged that the study of IRP without imported means of production is overly restrictive. However, this assumption is necessary to represent Shaikh's theory in its purest form, i.e., without corrections for non-tradable commodities and with *no imported means of production.* One way to accommodate this assumption is to posit that countries produce in autarchy and are suddenly open to free trade. For further insight, see also (Martínez-Hernández, 2017, fn 3, p. 7)

[9] See Smith (1994, ch. VI), Pasinetti (1973, §4), and Shaikh (1984, §IV).

indirect labor costs and profits per unit of commodity j. Appendix A.1 provides a formal derivation. Consequently, (2) can be expressed *equivalently* as:

$$p_j = \Omega_j + \Pi_j = \Omega_j(1 + \Psi_j) \quad \text{for} \quad j \in N, \tag{3}$$

where $\Omega_j \equiv \omega_j + \omega_j^I$ are the *total* or vertically integrated unit labor costs, $\Pi_j \equiv \pi_j + \pi_j^I$ are the total unit profits, and $\Psi_j \equiv \frac{\Pi_j}{\Omega_j}$ are the total profits-wages ratios, the most important variable in this paper.

We can express every Ψ_j and $(1 + \Psi_j)$ as the weighted average of the direct profits-wages ratios of all industries, ψ_j:

$$\Psi_j = \sum_{i=1}^{n} \psi_i m_{ij} \quad \text{for} \quad j \in N, \tag{4}$$

$$(1 + \Psi_j) = (1 + \sum_{i=1}^{n} \psi_i m_{ij}) = \sum_{i=1}^{n}(m_{ij} + \psi_i m_{ij}) = \sum_{i=1}^{n}(1 + \psi_i) m_{ij} \text{ for } j \in N, \tag{5}$$

where $m_{ij} \geqslant 0$ for $i, j \in N$ are the weights and $\sum_{i=1}^{n} m_{ij} = 1$. This weighting system condenses the process of vertical integration mentioned above. Equations (4)-(5) show the relevance of the properties of all the direct profits-wages ratios ψ_j and weights m_{ij} as well as their interaction for each total profits-wage ratio Ψ_j. Equation (4) is an alternative decomposition to that in Shaikh (1984, p. 65-9). Appendices A.2 and A.3 provide a detailed presentation and comparison of both decompositions, whereas A.4 provides some statistical properties of (4)-(5).

From equation (3) we can express relative prices of commodities j and k as

$$\frac{p_j}{p_k} = \frac{\Omega_j}{\Omega_k} \cdot \frac{(1 + \Psi_j)}{(1 + \Psi_k)} \quad \text{for} \quad j, k \in N, \tag{6}$$

where $\frac{p_j}{p_k}$ are the national relative prices (NRP), $\frac{\Omega_j}{\Omega_k}$ are the relative total unit labor costs (RTULC), and $\frac{(1+\Psi_j)}{(1+\Psi_k)}$ are the relative total profits-wages ratios (RTPWR). Shaikh (2016, p. 386) calls (6) the fundamental equation of relative prices. However, note that price expressions (2) and (3) are *equivalent*, i.e., they contain the same information because no additional constraints have been imposed.

Let us now decompose Ψ_j into an economy-wide profits-wages relation and the industry-level deviations around it. The economy-wide profits-wages ratio is[10]

$$\psi \equiv \frac{\text{Total profits}}{\text{Total wages}} = \frac{\sum_{j=1}^{n} \text{Profits}_j}{\sum_{j=1}^{n} \text{Wages}_j}, \tag{7}$$

where $(1 + \psi)^{-1}$ is the economy-wide wage-share. Define the normalized $(1 + \psi_j)$ as:

[10] Let $\bar{\psi} \equiv \frac{1}{n} \sum_{i=1}^{n} \psi_i$ be the simple average of the ψ_j of all industries. In general $\psi \neq \bar{\psi}$.

$$\sigma_j \equiv \frac{1+\psi_j}{1+\psi} \quad \text{for} \quad j \in N. \tag{8}$$

If each industry has the same ψ_j, then $\psi_j = \psi$ and $\sigma_j = 1$ for $j \in N$. Based on definition (8), we can decompose (5) as:

$$(1+\Psi_j) = \sum_{i=1}^{n}(1+\psi_i)m_{ij} = (1+\psi)\sum_{i=1}^{n}\sigma_i m_{ij} \quad \text{for} \quad j \in N. \tag{9}$$

Uniformity in total profits-wages ratios, i.e., $\Psi_j = \Psi_k$ for $j, k \in N$, implies that $\sum_{i=1}^{n}\sigma_i m_{ij} = \sum_{i=1}^{n}\sigma_i m_{ik} = \sigma$.

With this decomposition, we can express relative prices (6) as

$$\frac{p_j}{p_k} = \frac{\Omega_j}{\Omega_k} \cdot \frac{(1+\psi)}{(1+\psi)} \cdot \frac{\sum_{i=1}^{n}\sigma_i m_{ij}}{\sum_{i=1}^{n}\sigma_i m_{ik}} \quad \text{for} \quad j, k \in N. \tag{10}$$

Of course, within one particular country each price has the same $(1+\psi)$, so $\frac{p_j}{p_k} = \frac{\Omega_j}{\Omega_k}\frac{\sum_{i=1}^{n}\sigma_i m_{ij}}{\sum_{i=1}^{n}\sigma_i m_{ik}}$. However, the importance of the explicit identification of the nation wide $(1+\psi)$ in (10) will payoff for the study of international relative prices (IRP).

Proposition 1 initiates the series of constraints that ensure the equality of relative prices and the RTULC involved in the production process.

Proposition 1 *Relative prices between two commodities $k, j \in N$ within an economy are equal to their relative total unit labor costs,*

$$\frac{p_j}{p_k} = \frac{\Omega_j}{\Omega_k}, \tag{11}$$

if and only if industries' total profits-wages ratios are equal, $\Psi_j = \Psi_k = \Psi$.

The proof is evident, but its meaning carries important implications for the explanation of IRP. First and most importantly, we need to stress that a *general* theory of relative prices equal to their RTULC results from a constraint in the relative total profits-wages ratios, not from the *special* case of zero profits in the economy, $\Psi_j = 0$ for $j \in N$, which produces $p_j = \Omega_j$, and, therefore, (11).[11] Secondly, the loose constraints on the m_{ij}'s make possible to find a weighting system for Proposition 1 to hold (for $\Psi_j = \Psi_k = \Psi$) even under heterogeneous *direct* profits-wages ratios. Therefore, uniformity in *direct* profits-wages ratios of all industries $\psi_j = \psi$ are *sufficient* conditions for (11).[12] Finally, note that if profits in each industry are

[11] See also Basu (2017, Section 4).

[12] This result contrasts with the case of linear production models, where $\frac{p_j}{p_k} = \frac{\Omega_j}{\Omega_k}$ if and only if there is uniform *direct* profits-wages ratios in every industry (e.g., Kurz and Salvadori, 1995, p. 112-113). Note also that because (11) does not require $\psi_j = \psi$, then it follows that $\Psi_j = \Psi$ in general does not have to equal the economy-wide profits-wages ratio ψ in (7). For $\Psi = \psi$ we need $\sum_{i=1}^{n}\sigma_i m_{ij} = 1$ for $j \in N$ in (5). Sufficient conditions are $\psi_j = \psi$.

also positive, then $p_j = \Pi_j(1 + \Psi_j^{-1})$ and

$$\frac{p_j}{p_k} = \frac{\Pi_j}{\Pi_k} \cdot \frac{(1 + \Psi_j^{-1})}{(1 + \Psi_j^{-1})} \quad \text{for} \quad j, k \in N. \tag{12}$$

Hence, the same constraint characterizing Proposition 1, i.e., $\Psi_j = \Psi_k = \Psi$, also yields a theory of NRP governed by relative total unit profits, $\frac{p_j}{p_k} = \frac{\Pi_j}{\Pi_k}$.

2.3 Market terms of trade (ToT) between two commodities

Let us distinguish the variables from the price system introduced in Section 2.2 for country $\alpha = A, B$. Consider the ToT between commodity j produced in country A and commodity k produced in country B:

$$\frac{p_j^A}{p_k^B} = \frac{\Omega_j^A + \Pi_j^A}{\Omega_k^B + \Pi_k^B} = \frac{\Omega_j^A}{\Omega_k^B} \cdot \frac{(1 + \Psi_j^A)}{(1 + \Psi_k^B)} = \frac{\Omega_j^A}{\Omega_k^B} \cdot \frac{1 + \psi^A}{1 + \psi^B} \cdot \frac{\sum_{i=1}^n \sigma_i^A m_{ij}^A}{\sum_{i=1}^n \sigma_i^B m_{ik}^B}, \tag{13}$$

where $\frac{\Omega_j^A}{\Omega_j^B}$ are the RTULC and $\frac{(1+\Psi_j^A)}{(1+\Psi_k^B)}$ are the RTPWR. In contrast to NRP (10), the ToT (13) involve information on the techno-distributive characteristics of all industries between *two different countries.* In particular, $\frac{(1+\Psi_j^A)}{(1+\Psi_k^B)}$ in (13), in contrast to $\frac{(1+\Psi_j)}{(1+\Psi_k)}$ in (6), depends on the wage shares of the two countries $\frac{1+\psi^A}{1+\psi^B}$.

Proposition 2 *The terms of trade between commodity j and k exported by countries A and B are equal to their relative total unit labor costs,*

$$\frac{p_j^A}{p_k^B} = \frac{\Omega_j^A}{\Omega_j^B} \quad \textit{for} \quad j, k \in N, \tag{14}$$

if and only if the total profits-wages ratios of commodity j in country A and commodity k in country B are equal, $\Psi_j^A = \Psi_k^B$ for $j, k \in N$.

Although Proposition 2 is evident, its implications are important because, as we will see in Section 3.2, the literature on Shaikh's theory of IRP has overlooked the fact that (14) requires the equality of two magnitudes which involve technical-distributional aspects *from two different economies.* Even if we assume within-country uniformity in the total profits-wages ratios, in general we get

$$\left.\frac{p_j^A}{p_k^B}\right|_{\Psi_j^B=\Psi^B}^{\Psi_j^A=\Psi^A} = \frac{\Omega_j^A}{\Omega_j^B} \cdot \frac{(1 + \Psi^A)}{(1 + \Psi^B)} = \frac{\Omega_j^A}{\Omega_j^B} \cdot \frac{(1 + \psi^A)}{(1 + \psi^B)} \cdot \frac{\sigma^A}{\sigma^B} \quad \text{for} \quad j, k \in N, \tag{15}$$

where $\sum_{i=1}^n \sigma_i^\alpha m_{ij}^\alpha = \sigma^\alpha$ and $\alpha = A, B$. This leads to the following particular case:[13]

Proposition 3 *Suppose uniform direct profits-wages ratios within countries, $\psi_j^A = \psi^A$ and $\psi_j^B = \psi^B$. Then,*

[13] Remember that $\psi_i = \psi$ for $i \in N$ implies that $\sigma_i = 1$ and $\sum_{i=1}^n \sigma_i m_{ij} = \sigma = 1$ for $j, k \in N$.

$$\left.\frac{p_j^A}{p_k^B}\right|_{\psi_j^B=\psi^B}^{\psi_j^A=\psi^A} = \frac{\Omega_j^A}{\Omega_j^B} \tag{16}$$

if and only if the wage shares in country A and B are equal, $1+\psi^A = 1+\psi^B$.

The most important implication of this section is that if Proposition 2 holds for any pair of (tradable) commodities produced in A and B, then it is implied that $\Psi_j^A = \Psi_k^B = \Psi$ for $j, k \in N$, that is, it implies *international uniformity in the total profits-wages ratios.*

2.4 The market (bilateral) real exchange rate (RER)

To study the ToT between nations we need to consider the commodity bundles involved in their trade. Let $d^A = \{d_1^A, \ldots, d_n^A\}$ be the export basket of country A to country B and $d^B = \{d_1^B, \ldots, d_n^B\}$ the opposite, where $d_j^\alpha \geqslant 0$ but $d_j^\alpha > 0$ at least for some $j \in N$ for $\alpha = A{,}B$. With these bundles let us construct an *additive index of relative prices*, which is a generalization of the ToT discussed in Section 2.3.[14] The production of these baskets generates a stream of total unit labor costs $\Omega_d^\alpha \equiv \sum_{j=1}^n \Omega_j^\alpha d_j^\alpha$ and total unit profits $\Pi_d^\alpha \equiv \sum_{j=1}^n \Pi_j^\alpha d_j^\alpha$, and has the associated total profits-wages ratios $\Psi_d^\alpha \equiv \frac{\Pi_d^\alpha}{\Omega_d^\alpha}$. Export price indices are then

$$P_d^\alpha \equiv \sum_{j=1}^n p_j^\alpha d_j^\alpha = \Omega_d^\alpha (1+\Psi_d^\alpha) = (1+\psi^\alpha) \cdot \sum_{j=1}^n \left(\Omega_j^\alpha \cdot d_j^\alpha \cdot \sum_{i=1}^n \sigma_i^\alpha m_{ij}^\alpha \right) \tag{17}$$

for $\alpha = A, B$. Let e be the exchange rate between currencies in country A and B, the units of which are $\frac{€}{\$}$.[15] Therefore, the RER is then

$$\frac{P^A e}{P^B} = e \cdot \frac{\Omega_d^A}{\Omega_d^B} \cdot \frac{(1+\Psi_d^A)}{(1+\Psi_d^B)} = e \cdot \frac{\sum_{j=1}^n \left(\Omega_j^A \cdot d_j^A \cdot \sum_{i=1}^n \sigma_i^A m_{ij}^A\right)}{\sum_{j=1}^n \left(\Omega_j^B \cdot d_j^B \cdot \sum_{i=1}^n \sigma_i^B m_{ij}^B\right)} \frac{(1+\psi^A)}{(1+\psi^B)}, \tag{18}$$

where $\frac{\Omega_d^A}{\Omega_d^B}$ is the RTULC and $\frac{(1+\Psi_d^A)}{(1+\Psi_d^B)}$ is the RTPWR between commodity bundles d_d^A and d_d^B. The RER (18) is affected by the techno-distributive characteristics of all industries in A and B, just like in the ToT in (13). But (18) involves a qualitatively new feature affecting IRP: the semipositive quantities d_j^α can generate any positive value for Ψ_d^α, irrespective of the ψ_j^α, ψ^α, and m_{ij}^α.

Let us now complete the set of constraints that make relative prices to equal to the RTULC involved in their production.

[14] In Section 2.3 we had $d^A = \{0, \ldots, d_j^A, \ldots, 0\}$, $d^B = \{0, \ldots, d_k^B, \ldots, 0\}$, and $d_j^A = d_k^B = 1$.

[15] If we want to express all values in terms of the currency of country B (in €), we then have to multiply all values in country A by e. The assumption of one price for domestic and foreign sales makes that the real exchange rate between country A and B coincide with their terms of trade.

Proposition 4 *The real exchange rate between countries A and B exporting commodity baskets* d^A *and* d^B *is equal to the relative total unit labor costs involved in their production,*

$$\frac{P^A e}{P^B} = e \cdot \frac{\Omega^A_d}{\Omega^B_d}, \tag{19}$$

if and only if the total profits-wages ratios associated with the production of commodities d^A *and* d^B *in countries A and B, respectively, are equal,* $\Psi^A_d = \Psi^B_d$*.*

Just like with Proposition 2, for arbitrary commodities baskets d^A and d^B Proposition 4 requires the coordination of the techno-distributive characteristics between two economies. For the RER, we also have the situation where homogeneity in total profits-wages ratios within countries ($\sum_{i=1}^{n} \sigma^\alpha_i m^\alpha_{ij} = \sigma^\alpha$ for $j \in N$) cannot remove the influence of the relative wage-shares:

$$\left.\frac{P^A e}{P^B}\right|^{\Psi^A_d = \Psi^A}_{\Psi^B_d = e \cdot \Psi^B} = e \cdot \frac{\Omega^A_d}{\Omega^B_d} \cdot \frac{\sigma^A}{\sigma^B} \cdot \frac{(1+\psi^A)}{(1+\psi^B)}. \tag{20}$$

In the extreme case of uniform *direct* ratios we have $\sigma^\alpha = 1$ for $\alpha = A, B$, so

Proposition 5 *Suppose uniform direct profits-wages ratios within countries,* $\psi^A_j = \psi^A$ *and* $\psi^B_j = \psi^B$*. Then,*

$$\left.\frac{P^A e}{P^B}\right|^{\psi^A_j = \psi^A}_{\psi^B_j = \psi^B} = e \cdot \frac{\Omega^A_d}{\Omega^B_d} \tag{21}$$

if and only if the wage shares in country A and B are equal, $\psi^A = \psi^B$*.*

The introduction of commodity baskets d^A and d^B introduces an additional factor that affects the relationship between IRP and RTULC, beyond the techno-distributive conditions that have been previously identified.

Proposition 6 *There are commodity baskets* $d^A = \{d^A_1, \ldots, d^A_n\}$ *and* $d^B = \{d^B_1, \ldots, d^B_n\}$ *such that*

$$\frac{P^A e}{P^B} = e \cdot \frac{\Omega^A_d}{\Omega^B_d} \tag{22}$$

irrespective of the variability in their direct and the total profits-wages ratios.

International homogeneity of total profits-wages ratios is a sufficient but not necessary condition for Proposition 4 —The commodity bundles (d^A, d^B) are equally relevant in theory. But if Proposition 4 holds for any commodity basket, then $\Psi^A_d = \Psi^B_d$ can only hold under international uniformity in the total profits-wages ratios.

2.5 Long-period positions

The decomposition of national and international relative prices in (10), (13), and (18) was an *accounting* exercise in market values for the economy presented in Section 2.1. Once we introduce *economic constraints* on the total unit labor costs and total unit profits, such as in Proposition 1 to 6, the relative prices $\frac{p_j}{p_k}$, $\frac{p_j^A}{p_k^B}$, and $\frac{P_d^A}{P_d^B}$ are no longer relative market prices and become theoretical relative prices.

One set of constraints typically considered in theories of IRP-RC consists of long-period positions.[16] Some of these positions are (1) homogeneous labor and a uniform wage rate w, (2) uniform profit rates $r_j = r$, and (3) production prices $\bar{p}_j$.

Under long-period positions, *national* relative market prices have as centers of gravity the relative production prices that determine their long-run behavior, $\frac{p_j}{p_k} \approx \frac{\bar{p}_j}{\bar{p}_k}$. In addition, the total unit labor costs become $\Omega_j = wv_j$, where v_j is the total labor embodied in one unit of commodity j. Uniform profit rates across industries make total profits per unit of output to be $\Pi_j = rK_j$, where K_j is the total capital advanced per unit of output valued at production prices. Hence, $\Psi_j = \frac{r}{w}\frac{K_j}{v_j}$ and the economy-wide direct profits-wages ratio ψ represents the rate of surplus value.

Under these constraints, the ToT (13) become

$$\frac{p_j^A}{p_k^B} \approx \frac{\bar{p}_j^A}{\bar{p}_k^B} = \frac{w^A v_j^A}{w^B v_k^B} \cdot \frac{1 + \frac{r^A}{w^A}\frac{K_j^A}{v_j^A}}{1 + \frac{r^B}{w^B}\frac{K_k^B}{v_k^B}}. \tag{23}$$

If we further assume that $r^\alpha = r$, then there will be international production prices. However, these constraints does not make $\frac{r}{w^A}\frac{K_j^A}{v_j^A} = \frac{r}{w^B}\frac{K_k^B}{v_k^B}$, i.e., $\frac{\bar{p}_j^A}{\bar{p}_k^B} = \frac{w^A v_j^A}{w^B v_k^B}$, so long-period IRP still depend on international technological and distributive conditions.

2.6 Dynamic counterpart of Propositions 1 to 5

The different relative prices (RP) reviewed in this section —i.e., (6), (13), (18), and (23)— have the same multiplicative structure: $RP = RTULC \cdot RTPWR$. This implies that $\widehat{RP} = \widehat{RTULC} + \widehat{RTPWR}$[17] and that there will be a dynamic counterpart of propositions 1 to 5. In each case, $\widehat{RP} = \widehat{RTULC}$ if and only if $\widehat{RTPWR} = 0$, that is, if and only if $\widehat{1 + \Psi_d^\alpha} = \widehat{1 + \Psi_d^\beta}$. In this case, $\frac{1+\Psi_d^\alpha}{1+\Psi_d^\beta}$ stay constant, even if $\Psi_d^\alpha \neq \Psi_d^\beta$, in contrast with propositions 1 to 5.[18]

[16] These constraints, which arise from the process of competition between workers and between capitals in commodity-producing economies under wage labor, affect the nature of the labor, the rates of return of labor and the capital advanced, and the characterization of prices. See Garegnani (1976), Foley (2011, 2016), and Cogliano (2018, 2023).

[17] For any variable that is a continuous function of time $x = x(t)$, define $\dot{x} \equiv \frac{d}{dt}x$ and $\hat{x} \equiv \frac{\dot{x}}{x}$.

[18] The same reasoning applies for the expected value $\mathrm{E}\left(\widehat{RP}\right) = \mathrm{E}\left(\widehat{RTULC}\right) \iff \mathrm{E}\left(\widehat{RTPWR}\right)$, and the stationarity of the variable, $\frac{RP}{RTULC} \sim I(0) \iff RTPWR \sim I(0)$.

3 The Theory of IRP based on 'Real Competition' and its Evaluation

3.1 The construction of the theory of IRP-RC

Shaikh argues that the *long-run* behavior of international relative prices (IRP) is the result of international capital competition, just like capital competition at the national level determines the long-run national relatie prices (NRP).[19] The competitive framework used to support this result is Shaikh's own theory of 'Real Competition' (RC) —hence IRP-RC.[20] The theory of RC results from the addition of further constraints on long-period positions. Based on this framework, he develops a series of arguments to construct hypotheses that attempt to establish the main thesis of the theory: *for any pair of internationally tradable commodity bundles, the long-run behavior of their relative market prices is determined exclusively by their associated relative total unit labor costs (RTULC).*

3.1.1 Regulating capitals and their production prices

The theory of RC modifies the long-period positions by adding one layer of determinations that avoids the homogeneity of conditions of production and reproduction of capitals within industries that was assumed in Section 2.1, but retains the competitive process and outcomes for a subset of capitals, which he calls regulating capitals.[21]

Within each industry, different types of factors create cost heterogeneity among capitals. Capitals will operate under different production conditions. Only some of these conditions will be reproducible for the newly arriving capitals. Among these capitals producing under reproducible conditions, there will be a subset that operates at the lowest unit cost. These capitals are called the 'regulating capitals' of the industry and are able to set the price of the commodity which, according to the law of one price, becomes the center of gravity of the rest of the prices set by the spectrum of capitals within this industry. The tendency to the homogeneity of prices combined with the heterogeneity of costs produces a spectrum of profit rates within this industry.

In the search for the highest rates of return, the competition between the regulating capitals of the various industries produces a tendency towards the equalization

[19] 'This paper develops and tests a long run theory of the exchange rate based upon a classical approach to the theory of competition ... It is first applied to competition within one "nation" ... and then extended to the multi-currency case' (Shaikh, 1991, p. 2). See also (Shaikh, 1999, p. 12). Sarich (2006, pp. 470, 476-80) arrives to the same interpretation in providing an alternative explanation for the empirical evidence on the relationship between the RER and relative costs of production within the Harrod-Balassa-Samuelson effect literature.

[20] See Shaikh (1991, §I & II; 1995, pp. 70-1; 1999, pp. 1-3; 2002, pp. 6-7; 2016, §7.IV).

[21] See Shaikh (2016, §7.II & 7.IV).

of their rates of profit. From this process it emerges a set of production prices for the regulating capitals, which constitute the new centers of gravity of market prices.

It is then argued that IRP are 'determined by the equalization of profit rates across international regulating capitals' (Shaikh, 1999, p. 0).[22] That is, the long-run behavior of international market relative prices is determined by the *relative production prices of the regulating capitals* in the industries producing these commodities, $p_j^A/p_k^B \approx \bar{p}_j^A/\bar{p}_k^B$, where $(\bar{p}_j^A, \bar{p}_k^B)$ are the production prices of the international regulating capitals.

3.1.2 The hypotheses constraining the relative production prices of the regulating capitals

The thesis of the theory of IRP-RC *requires one more layer of determinations*, this time constraining the production prices of the regulating capitals to have as a backbone or unique 'long-run determinant' the RTULC, $p_j^A/p_k^B \approx \bar{p}_j^A/\bar{p}_k^B \approx \Omega_j^A/\Omega_k^B$. It is argued that these constraints arise from the forces of competition between regulating capitals.

Shaikh characterizes this *long-run behavior* as a 'gravitation in the orbital sense' (1991, p. 1). Sometimes he means an oscillatory orbiting (1991, p. 7) —compatible with medium-term cycles (2016, p. 525)— or turbulent gravitation (2016, p. 530) of the IRP around the RTULC. Other times he treats $RTPWR = \frac{IRP}{RTULC}$ as a stationary process (2002, p. 9) with a mean-reversion (2016, p. 525) or, alternatively, he considers $(1 + \Psi_j^A/1 + \Psi_k^B)$ as a 'disturbance' term centered around one (e.g., Shaikh 2016, p. 518). Based on this, there are no reasons to expect any bias in the difference between p_j^A/p_k^B and Ω_j^A/Ω_k^B, or equivalently, between Ψ_j^A and Ψ_k^B, in the long-run.

To obtain $p_j^A/p_k^B \approx \Omega_j^A/\Omega_k^B$, the theory of IRP-RC proceeds in two steps —one for the ToT and another for the RER. First, it advances the *hypothesis* that the long-run behavior of NRP is determined by their RTULC, $p_j/p_k \approx \Omega_j/\Omega_k$. Then, it is *argued* that this constraint on NRP implies that the long-run ToT of any two tradable commodities is also determined by the RTULC associated in their production, $p_j^A/p_k^B \approx \Omega_j^A/\Omega_k^B$. Second, and building on the previous result, it advances the *hypothesis* that $p_j^A/p_k^B \approx \Omega_j^A/\Omega_k^B$ and then it is *argued* that this constraint on the ToT implies that the long-run RER of any two tradable commodity *bundles* is also determined by the RTULC associated in their production, $P_d^A/P_d^B \approx \Omega_d^A/\Omega_d^B$. Given the information in Propositions 1 and 2, if we replace the equalities (=) by approximations (≈), then each of the hypotheses can be equivalently expressed as:

[22] Most studies in the literature share this view. For instance, '[i]n the determination process of long-run real exchange rates [the] leading dynamic is the profit rate equalization' (Ersoy, 2010, p. 16).

Hypothesis 1 ($\mathcal{H}_1$) *The long-run behavior of relative prices of commodities j and k is determined by their relative total unit labor costs,*

$$\frac{p_j}{p_k} \approx \frac{\Omega_j}{\Omega_k} \quad \textit{for all} \quad j, k \in N, \tag{24}$$

or, the total profits-wages ratios of industries j and k are sufficiently close,

$$\Psi_j \approx \Psi_k \approx \Psi, \quad \textit{for all} \quad j, k \in N. \tag{25}$$

Hypothesis 2 ($\mathcal{H}_2$) *The long-run behavior of the terms of trade of commodities j and k exported by countries A and B is determined by their relative total unit labor costs,*

$$\frac{p_j^A}{p_k^B} \approx \frac{\Omega_j^A}{\Omega_k^B} \quad \textit{for some} \quad j, k \in N, \tag{26}$$

or, the total profits-wages ratios of industry j in country A and industry k in country B are sufficiently close,

$$\Psi_j^A \approx \Psi_k^B, \quad \textit{for some} \quad j, k \in N. \tag{27}$$

From this structure, it is easy to see that the theory of IRP-RC relies on the theory of the long-run NRP. Overall, the logical strength of the thesis depends on the *soundness* in the construction of Hypothesis 1 and 2 and the *effectiveness* of these hypotheses in deriving the results $p_j^A/p_k^B \approx \Omega_j^A/\Omega_k^B$ and $P_d^A/P_d^B \approx \Omega_d^A/\Omega_d^B$, respectively. Figure 1 provides a schematic representation of the theory by identifying (1) the hypotheses ($\mathcal{H}$), (2) the constraints supporting the hypotheses ($\mathcal{C}$), and (3) the argumentation linking the constraints/hypotheses to the desired results ($\mathcal{A}$). The dependence of the theory of IRP-TC on the theory of NRP is then represented with the gray boxes and lines. The top black boxes and lines link the hypotheses with the desired results on IRP.

3.2 The evaluation of the theory

We now provide an evaluation of the *soundness* of the hypotheses (Section 3.2.1), the *effectiveness* of the hypotheses (Section 3.2.2) and, finally, of the role played by the *forces of competition* in deriving the result that the long-run behavior of IRP is determined by their associated RTULV (Section 3.2.3).

3.2.1 The *soundness* of Hypothesis 1

The argument that $p_j/p_k \approx \Omega_j/\Omega_k$ within a country implies $p_j^A/p_k^B \approx \Omega_j^A/\Omega_k^B$ transfers some properties of *domestically* produced commodities to *internationally* produced commodities. It was originally stated in Shaikh (1991, p. 6; 1995, pp. 70-71)

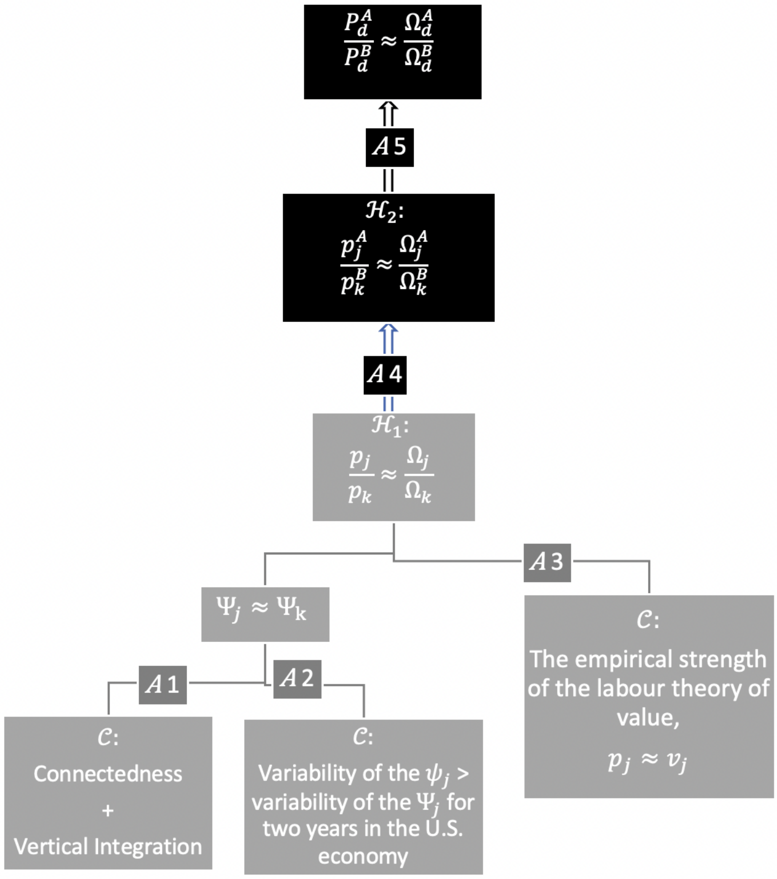

Figure 1: Structure of Shaikh's theory of international relative prices. $\mathcal{C}$, $\mathcal{A}$, and $\mathcal{H}$ refers to constraint, argument, and hypothesis, respectively.

and has since been used as the basis for empirical studies on the theory of IRP-RC.[23] This hypothesis is supported by three arguments pertaining the existence of forces in the economy that make ${}^{p_j}\!/_{p_k} \approx {}^{\Omega_j}\!/_{\Omega_k}$. In Argument 1 and 2 these forces define explicit constraints on the Ψ_j in such a way that ${}^{(1+\Psi_j)}\!/_{(1+\Psi_k)}$ can be considered a 'disturbance' term close to one. In Argument 3 the forces manifest themselves directly on the prices, so that ${}^{p_j}\!/_{p_k} \approx {}^{\Omega_j}\!/_{\Omega_k}$. Since Proposition 1 states that ${}^{p_j}\!/_{p_k} = {}^{\Omega_j}\!/_{\Omega_k} \iff \Psi_j = \Psi_k$, arguments 1-3 are mutually reinforcing. We will now consider each of them in detail and give an evaluation.

[23] 'Within a nation, the relative prices of products can be well approximated by the relative total ... real unit labor costs ... In an international context, this same principle translates into the proposition that the relative common-currency prices of any two goods in the world market are regulated by the total real unit labor costs of the countries ...' (Antonopoulos, 1999, p. 58). See also Ge (1993, p. 266), Guerrero (1995, §4.3.A.a.1), Martínez-Hernández (2010, pp. 66-67), and Moreno-Rivas (2018, p. 25) provide similar arguments. See also the transition from NRP to IRP in Boundi-Chraki (2020, p. 2), Boundi-Chraki and Perrotini-Hernández (2021, p. 164), and Martínez-Hernández (2017, pp. 6-7), respectively. Fevereiro (2019), while considering the total profits-wages ratios between different countries (Ψ_j^A, Ψ_k^B), argues that $\Psi_j \approx \Psi_k$ implies $\Psi_j^A \approx \Psi_k^B$.

Argument 1 ($\mathcal{A}_1$) *In a highly connected economy, the process of vertical integration makes the variability of the total profits-wages ratios* Ψ_j *to be sufficiently small, compared to whatever variability is in the direct profits-wages ratios* ψ_j*, so that* $\Psi_j \approx \Psi_k$ *for* $j, k \in N$*.*

Argument 1 was first presented by Shaikh (1984, §4) in the literature on the classical theory of prices and the labor theory of value.[24] It was later introduced in the literature on the theory of IRP-RC by Shaikh (1991, 1995) and a large part of the literature has relied on these arguments ever since.[25] The argument develops as follows: in every economy there is a certain variability in the direct profits-wages ratios ψ_j. The total profits-wages ratios $\Psi_j = \sum_{i=1}^{n} \psi_j m_{ij}$ are weighted averages of these ψ_j. It is then argued that, in a highly connected economy, in the sense of a dense interindustry input-output network, the process of vertical integration makes each Ψ_j to have appropriate weights m_{ij} such that the variability of Ψ_j is sufficiently small in order for $\Psi_j \approx \Psi$ and therefore $p_j/p_k \approx \Omega_j/\Omega_k$ for $j \in N$.

There are two problems with this theoretical argument: The first one is the belief that the process of vertical integration of the industries, such as that which expresses prices in (2) as (3), adds a constraint that affects the properties of relative prices. In Section 2.2 it was shown that prices in terms of direct and vertical integrated industries are *equivalent*. If it happens that $\Psi_j \approx \Psi_k$ this is not because of a strong or weak process of vertical integration. As shown in (4) and Appendix A.4, if $\Psi_j \approx \Psi_k$ it is because of particular statistical properties of the direct profits-wages ratios ψ_j, of the weights m_{ij}, and their relation.[26]

The second problem with Argument 1 concerns the contribution of interindustry *connectedness* for $\Psi_j \approx \Psi$ for $j \in N$. General configurations of the interindustry

[24] And constantly reproduced in this literature: 'Now it is very important to recognise that the influence of [the $\frac{1+\Psi_j}{1+\Psi_k}$] is likely to be rather small, because its elements depend on the degree to which different convex combinations of direct profit–wage ratios differ from each other. As a consequence, even large variations in sectoral profit–wage rates are reduced to small ones in the corresponding integrated ratios. Therefore, equation (24) is a modified law of value, with [the $\frac{1+\Psi_j}{1+\Psi_k}$] containing some kind of probably negligible disturbance factors. If there is any transformation problem, it is most likely moderate. But this is an empirical question' (Fröhlich, 2012, p. 1112).

[25] For instance: 'For the reasons which have been explained by Shaikh, ... the second term can be viewed as a disturbance term so that than approximation to the above equation [$\frac{p_j}{p_k} \approx \frac{\Omega_j}{\Omega_k}$] is derived' Ge (1993, p. 259). See also (Guerrero, 1995, §4.3.A.a.1), Martínez-Hernández (2017, p. 5), and Boundi-Chraki and Perrotini-Hernández (2021, p. 164).

[26] In support of Argument 1, Shaikh (2016, pp. 387-8) argues that $\frac{1+\Psi_j}{1+\Psi_k} \approx 1$ even if Ψ_j differs considerably from Ψ_k. Shaikh construct an example where $\Psi_j = 0.40$ and $\Psi_k = 0.20$, that is, where Ψ_j doubles Ψ_k, and obtains $\frac{1+\Psi_j}{1+\Psi_k} = 1.16$, concluding that $\frac{p_j}{p_k}$ deviates from $\frac{\Omega_j}{\Omega_k}$ only by 16%. However, this result depends on the small value of Ψ_j and Ψ_k which makes $\frac{1+\Psi_j}{1+\Psi_k} \approx 1$, even if Ψ_j *triples* Ψ_k (for example $\frac{1+0.21}{1+0.07} \approx 1.13$). If instead Ψ_j doubles Ψ_k but with higher magnitudes, e.g., $\Psi_j = 2$ (VI profits doubles total wages) and $\Psi_k = 1$, then $\frac{1+\Psi_j}{1+\Psi_k} = \frac{3}{2} = 1.5$, so $\frac{p_j}{p_k}$ deviates from $\frac{\Omega_j}{\Omega_k}$ by 50%.

input-output network do not systematically affect Ψ_j. Consider the Leontief-Sraffian models. It is well known that for almost every configuration of the input-coefficient matrix, the matrix containing the information of the 'interindustry connectedness', we have ${}^{p_j}/_{p_k} = {}^{\Omega_j}/_{\Omega_k}$ or $\Psi_j = \Psi_k$ only by a fluke (see, Sraffa 1960, ch. III; Pasinetti 1977, pp. 82-4).[27] In fact, one of the factors that contributes to the low likelihood of a sufficiently small variability in the Ψ_j is precisely that industries use as means of production commodities produced in other industries. The only configuration of interindustry commodity flows which produces $\Psi_j = \Psi_k = \Psi$ is when the *labor vector* is proportional to the Perron-Frobenius eigenvector of the input matrix. That is, $\Psi_j = \Psi_k$ does not depend solely on interindustry connectedness.

Argument 2 ($\mathcal{A}_2$) *For the U.S economy, the standard deviation and the coefficient of variation of industries' total profits-wages ratios are considerably smaller than the same variability indicators for the direct profits-wages ratios, so that* $\Psi_j \approx \Psi_k$ *for* $j, k \in N$.

Shaikh (1984, p. 77; 2016, p. 388) compares the standard deviation and coefficient of variation of industries' direct and total profits-wages ratios and capital intensities for the U.S. economy for the years 1947 and 1998 and reports that the variability of the total ratios is considerably lower than that of the direct ratios, thus arguing that $\Psi_j \approx \Psi_k$ so that $\frac{p_j}{p_k} \approx \frac{\Omega_j}{\Omega_k}$ for this economy. Argument 2 is then empirical. The literature on the theory of IRP-RC has extrapolated this property from two years of the U.S. economy for their economies under study. Up to the authors' best knowledge, this is the only available information on the statistical properties of industrial profits-wages ratios or capital intensities.[28]

The first problem with Argument 2 is that our critique of Argument 1 implies that the reduced variability of the Ψ_j *vis-a-vis* the ψ_j cannot be used as evidence in favor of sufficient $\Psi_j \approx \Psi_k$ for ${}^{p_j}/_{p_k} \approx {}^{\Omega_j}/_{\Omega_k}$. The properties of relative prices are the same either expressed in terms of direct or vertically integrated industries. The variability of the Ψ_j affects ${}^{p_j}/_{p_k}$. Yet, none of the authors in the literature has prove that the properties of relative prices depend on the reduced variability of the Ψ_j *compared to* ψ_j.

The second problem with Argument 2 is precisely the lack of knowledge regarding the statistical properties of the profits-wages ratios. The literature on IRP-RC

[27] Schefold (1976) and Bidard and Salvadori (1995) show that by marginally constraining the input matrices and the labor vectors (in order to produce what they call regular systems), price systems then have the property that for any n different profit rates there will be n linearly independent price vectors, so there is nothing that ensures that *in general* ${}^{p_j}/_{p_k} = {}^{\Omega_j}/_{\Omega_k}$ or $\Psi_j \approx \Psi_k$.

[28] Shaikh (2016, Table 9.5, p. 388) reports four numbers with the averages of the *direct* profits-wages ratios, one for each set of countries (the EU, the U.S., Japan, and Canada) and for the period 1960-2011. It is not clear whether these four country-years averages are for the economy-wide ratios or for their industries.

typically examines economies with significant technical and distributional discrepancies (e.g., Spain-Germany, Mexico-U.S.). And yet, there is no empirical study of ψ_j, m_{ij}, and Ψ_j. Argument 2 cannot be considered a stylized fact supporting the constraint '$\Psi_j \approx \Psi_k$ for any market economy'.

Argument 3 ($\mathcal{A}_3$) *Due to the labor theory of value, market prices p_j are well approximated by the total quantities of labor v_j, so that $\frac{p_j}{p_k} \approx \Omega_j/\Omega_k$.*

Part of the literature on the theory of IRP-RC relies (completely or as a complement) on a version of the Ricardian or Marxian theory of value.[29] It is argued that the centers of gravity of relative market prices are given by their relative labor values, that is, by the relative quantities of total labor involved in the production of the commodities, such that $p_j/p_k \propto \frac{v_j}{v_k} = \Omega_j/\Omega_k$.[30] This view is also taken in studies of international trade and unequal exchange based on the theory of RC (e.g., Seretis and Tsaliki 2016, pp. 444-5; Tsaliki et al. 2017, pp. 1052-5). Argument 3 differs from Argument 1 and 2 in that the former does not rest on an explicit constraint on the Ψ_j, but it rather rests either on the axiom $p_j/p_k \approx v_j/v_k$ from the operation of the Ricardian or Marxian 'law of value'[31] or as a hypothesis sustained by empirical evidence on the relationship between market, direct, and production prices.[32]

The literature review by Basu (2017, §4) organizes these empirical exercises in two classes: those that conduct the regression $\ln \frac{p_j}{p} = \alpha + \beta \ln \frac{v_j}{v}$ and those that evaluate the proximity of vectors $(\frac{p_1}{v_1}, \frac{p_2}{v_2}, \ldots, \frac{p_n}{v_n})$ and $(1, 1, \ldots, 1)$ using different distance measures.[33] One problem with this approach is that there is no evaluation

[29] 'If we apply Ricardo's theory of price determination, as developed by Pasinetti ... we cannot assume that the profit rate is zero; neither are we estimating natural prices with a positive uniform profit rate [and] uniform capital–labour ratios across sectors ... we are neither estimating natural prices between countries as if they were regions within one country ... The proposition is that ... we are calculating vertically integrated unit labour costs (VIULC). But according to Ricardo, these average VIULC will determine average prices in each economy and, therefore, the ratio of the two countries' VIULC determines the real exchange rate between their currencies' (Ruiz-Nápoles, 2004, p. 76). See also Ersoy (2010, p. 16), Martínez-Hernández (2017, p. 6), Féliz and Pedrazzi (2019, p. 60), Boundi-Chraki (2019, pp. 125), and Poulakis and Tsaliki (2023a, p. 608-9; 2023b, p. 9). Aglietta and Oudiz (1984, p. 94) *define* the RER as the relative unit labor costs based on the Ricardian theory of value.

[30] For the Ricardian version the substance of labor-values is 'labor time' whereas for the Marxian version 'socially necessary labor time'. In their mathematical modeling, the v_j are re-scaled such that the sum of prices corresponds to the sum of values. For an alternative understanding of the labor theory of value see Foley (1986, ch. 2; 2011; 2016).

[31] e.g., Ruiz Nápoles (2004, pp. 76-7; 2010, pp. 18-21), Martínez-Hernández (2010, pp. 65-6), and Boundi-Chraki (2019, pp. 124-5).

[32] Mariolis and Tsoulfidis (2016, chap. 3) and Shaikh (2016, chap. 8, sect. V, VI, VIII, IX) provide a thorough exposition of the methodology. Cheng and Li (2019) provides a survey of the literature. Ochoa (1984) conducts the most detailed study and Işıkara and Mokre (2021) the most extensive one.

[33] The average deviations between individual market and direct prices is typically small across different economies and through the use of different models and metrics —typically less than 20%.

of the pairs of relative market and direct prices $(p_j/p_k, v_j/v_k)$ for $j, k \in N$, which would involve the study of $\frac{n(n-1)}{2}$ relative prices (Fröhlich, 2012, p. 1110). In studying only the relationship between p_j and v_j it is sustained that $p_j/p_k \approx \Omega_j/\Omega_k$ for $j, k \in N$.

3.2.2 The *effectiveness* of Hypothesis 1 and 2 deriving the results $p_j^A/p_k^B \approx \Omega_j^A/\Omega_k^B$ and $P_d^A/P_d^B \approx \Omega_d^A/\Omega_d^B$

The literature on the theory of IRP-RC sustains that Hypothesis 1, and therefore Argument 1 to 3, *imply*, with no further constraints, that the long-run behavior of the ToT (Argument 4) and RER (Argument 5) is governed by the RTULC involved in the production of the tradable commodity bundles composing the IRP.

Argument 4 ($\mathcal{A}_4$) *The determination of the long-run behavior of any national relative prices p_j/p_k by their relative total unit labor costs Ω_j/Ω_k (i.e., $\mathcal{H}_1$) implies that the long-run behavior of the terms of trade between any two commodities exported by two countries p_j^A/p_k^B is determined by their relative total unit labor costs Ω_j^A/Ω_k^B.*

Argument 4 transfers some properties of NRP between any two commodities to the ToT of any two commodities but produced in different countries. Given Proposition 1 and 2, Argument 4 can be stated as $p_j/p_k \approx \Omega_j/\Omega_k \rightarrow p_j^A/p_k^B \approx \frac{\Omega_j^A}{\Omega_k^B}$ or $\Psi_j^\alpha \approx \Psi_k^\alpha \approx \Psi^\alpha \rightarrow \Psi_j^A \approx \Psi_k^B$. If the approximations are replaced with equal signs, then Argument 4 can be stated as Proposition 1 implies Proposition 2.

It is then obvious that Hypothesis $\mathcal{H}_1$ has an efficiency problem and Argument 4 is problematic: $\Psi_j^A = \Psi^A$ and $\Psi_j^B = \Psi^B$ do not imply that $\Psi^A = \Psi^B$. The problem does not originate in the degree of heterogeneity of the profits-wages ratios *within* countries. Even if we assume homogeneity of the direct or total profits-wages ratios within countries, Equation (15) and Proposition 3 show that the ToT still depends on the relative wage shares or rates of exploitation of countries A and B, $p_j^A/p_k^B = \Omega_j^A/\Omega_k^B \cdot 1+\psi^A/1+\psi^B$.[34] The only configuration of these ratios that supports Argument 4 is $\Psi_j^A \approx \Psi_k^B \approx \Psi$. That is, $p_j^A/p_k^B \approx \Omega_j^A/\Omega_k^B$ for each pair of tradable commodity implies *international uniformity* in total profits-wages ratios.

This required assumption of international homogeneity of total profits-wages ratios across industries and countries contradicts one of the pillars of the theory

The results are robust for (1) a wide sample of countries with different development levels, (2) different scalar metrics to measure the deviations (e.g., mean absolute deviations, mean-weighted absolute deviations, among others) (see Mariolis and Tsoulfidis 2016, ch. 4; Shaikh 2016, ch. 9, §IV), and (3) different Leontief-Sraffian models (with and without fixed capital and turnover rates, and with ex-post or ex-ante wages).

[34] The research program of Martínez-González and Valle-Baeza has documented the international differences in rates of surplus value. See inter alia, Martínez-González (2005), Martínez-González and Valle-Baeza (2023), and Martínez-González et al. (2019). See also Section 5 for more on this.

of IRP-RC, namely, the existence of non-trivial techno-distributive international differences:[35]

> [I]t is an essential feature of our analysis that production conditions and real wages are assumed to differ across countries. Neoclassical theory tends to assume that production functions are similar across countries ... Yet nothing could be further from the empirical facts ... Dollar, Wolff, and Baumol (1988) find that for any given industry, productivity varies substantially across countries ... such international productivity variations are largely explained by corresponding *variations in real capital-labor ratios* ... countries with higher productivity and/or capital-labor ratios in one industry tend to have higher measures in all industries ... and countries with higher productivity in a given industry tend to also have higher wages in the same industry ... On this basis we assume that real wages and technology are determined locally in each nation (Shaikh, 1995, pp. 68-9, our emphasis).

Authors in the literature posit the existence of these international differences in technology and distribution.[36] In light of these alleged international differences, it is difficult to maintain $\Psi_j^A \approx \Psi_k^B \approx \Psi$ for $j, k \in N$.

The dependence of IRP on *international* techno-distributive differences has escaped the attention in the literature.[37] One consequence of this omission is the absence of a comprehensive understanding of the statistical characteristics associated with the profits-wages ratios within the existing literature.[38]

Argument 5 ($\mathcal{A}_5$) *Because what happens to individual relative prices happens to relative price indices, then* $\frac{p_j^A}{p_k^B} \approx \frac{\Omega_j^A}{\Omega_k^B}$ *implies that* $\frac{P_d^A}{P_d^B} \approx \frac{\Omega_d^A}{\Omega_d^B}$.

The study of the RER introduces commodity baskets d^A and d^B for the construction of price indices P_d^A and P_d^B. Argument 5 can be stated, if approximations are substituted for equalities, as Proposition 2 implies Proposition 4,[39] that is, $\Psi_j^A \approx$

[35] See also Shaikh (1980, §II.2, specially footnote 27).

[36] 'With regard to international competition ... one would expect that techniques of production of any World Industry, where individual firms are spread out through various countries, will vary from one nation to another as well' (Martínez-Hernández, 2010, p. 64). See also Ruiz-Nápoles (2004, p. 73), Góchez-Sevilla and Tablas (2013, p. 7), and Boundi-Chraki (2019, p. 120).

[37] An expression similar to (13), where all components of IRP are explicitly stated before constraints are introduced, only appeared in the literature only until Góchez-Sevilla and Tablas (2013, p. 31) and Shaikh (2016, p. 518). Previously, it was assumed that NRP close to their RTULC must imply the same faith for IRP. For instance, '[i]f we let p denote unit price, and v denote the unit vertically integrated labor cost of the regulating producer, then for any two industries within a nation we may write ... $(p_i/p_j) \approx (v_i/v_j)$ ... The same principle may be applied on an international scale, modified only to take into account the distinction between national currencies' (Shaikh and Antonopoulos, 2012, p. 209). See also Poulakis and Tsaliki (2023a, p. 608-9; 2023b, p. 9-10).

[38] Fevereiro (2019, §3) is the only exception —with his study of national wage-shares.

[39] 'In keeping with the empirical results of chapter 9 (see equation 9.5), we can also assume that the disturbance term is relatively small so that $[1 + \Psi_d^A / 1 + \Psi_d^B \approx 1]$' (Shaikh, 2016, p. 518). Some studies even assume that $P_d^A / P_d^B \approx \Omega_d^A / \Omega_d^B$ derives from the approximate verification of the conditions of Proposition 1 (e.g., Boundi-Chraki and Perrotini-Hernández 2021, p. 164).

$\Psi_k^B \rightarrow \Psi_d^A \approx \Psi_d^B$.

This argument holds only in very restrictive cases. On the one hand, for a given commodity basket, the general validity of Argument 5 requires the assumption of international uniformity in the total profits wages ratios, $\Psi_j^A \approx \Psi_j^B \approx \Psi$ for $j \in N$. On the other hand, Proposition 6 shows that it is always possible to find commodity baskets which bring uniformity to the index of total profits-wages ratios *for any set of total profits-wages ratios* involved in the construction of the price indices. This 'quantity-side' of IRP determination has been missing in the literature.[40]

3.2.3 On the role of competition constraining production prices

We conclude the evaluation of the theory of IRC-RC by considering the role played by the forces of competition given either by the long-period positions or by the constraints of the theory of RC. The process of mobility of all capitals or of the regulating capitals seeking the highest rates of return and producing a tendency towards the equalization of their rates of profit *cannot* produce the equality of the total profits-wages ratios of any two commodity baskets. Under long-period positions, the ratios $\Psi_j = \frac{r}{w}\frac{K_j}{v_j}$ depend on distributional and technical dimensions of the economy. The literature has not shown how capital mobility modifies $\frac{K_j}{v_j}$ such that $\Psi_j = \Psi_k$ for $j \in N$. Actually, the theoretical developments on the formation of an average profit rate and production prices usually take as given the technique of production. Another process of competition, i.e., technical change, could definitely constrain the ratios Ψ_j by affecting the capital intensities $\frac{K_j}{v_j}$. However, technical change at the capital level is more likely to increase rather than reduce the variability of the profit rates. Arguments 1-3 from the theory of IRP-RC do not depend on the emergence of production prices.[41]

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[40] To see the problems of Argument 5, let us consider a simple example. Suppose that $N = \{1, 2, 3, 4\}$, $d^A = \{d_1^A, d_2^A, 0, 0\}$ and $d^B = \{0, 0, d_3^B, d_4^B\}$, that is, A exports commodities 1 and 2 to B and B exports commodities 3 an 4 to A. Because $\frac{P_{\{1,2\}}^A}{P_{\{3,4\}}^B} = \frac{p_1^A d_1^A + p_2^A d_2^A}{p_3^B d_3^B + p_4^B d_4^B} = \frac{(\Omega_1^A d_1^A + \Pi_1^A d_1^A) + (\Omega_2^A d_2^A + \Pi_2^A d_2^A)}{(\Omega_3^B d_3^A + \Pi_3^B d_3^A) + (\Omega_4^B d_4^B + \Pi_4^B d_4^B)} = \frac{\Omega_d^A}{\Omega_d^B} \cdot \frac{1 + \left(\delta_1^A \Psi_1^A + \delta_2^A \Psi_2^A\right)}{1 + \left(\delta_3^B \Psi_3^B + \delta_4^B \Psi_4^B\right)}$, the RER is then $\frac{P_{\{1,2\}}^A}{P_{\{3,4\}}^B} = \frac{\Omega_d^A}{\Omega_d^B} \cdot \frac{1 + \Psi_d^A}{1 + \Psi_d^B}$, where $\Omega_d^A = \Omega_1^A d_1^A + \Omega_2^A d_2^A$, $\Omega_d^B = \Omega_3^B d_3^B + \Omega_4^B d_4^B$, $\Psi_d^A \equiv \delta_1^A \Psi_1^A + \delta_2^A \Psi_2^A$, $\Psi_d^B \equiv \delta_3^B \Psi_3^B + \delta_4^B \Psi_4^B$, $\delta_j^A \equiv \frac{\Omega_j^A d_j^A}{\Omega_1^A d_1^A + \Omega_2^A d_2^A}$ and $\delta_k^B \equiv \frac{\Omega_k^B d_k^B}{\Omega_3^B d_3^B + \Omega_4^B d_4^B}$, and $\delta_1^A + \delta_2^A = \delta_3^B + \delta_4^B = 1$. Suppose in addition that Hypothesis 1 holds for the trade between commodities 1 and 3 and 2 and 4, i.e., that $p_1^A/p_3^B \approx \Omega_1^A/\Omega_3^B$ and $p_2^A/p_4^B \approx \Omega_2^A/\Omega_4^B$, so $\Psi_1^A \approx \Psi_3^B$ and $\Psi_2^A \approx \Psi_4^B$. From the equation for $P_{\{1,2\}}^A/P_{\{3,4\}}^B$, we can see that in general is not possible to have $P_{\{1,2\}}^A/P_{\{3,4\}}^B \approx \Omega_d^A/\Omega_d^B$ because we cannot say that *in general* $\delta_1^A \Psi_1^A + \delta_2^A \Psi_2^A \approx \delta_3^B \Psi_1^A + \delta_4^B \Psi_2^A$. We would need the special constraints of international uniformity in the ratios $\Psi_1^A \approx \Psi_2^A \approx \Psi_3^B \approx \Psi_4^B \approx \Psi$ or specific commodity baskets d^A and d^B such that the weights $(\delta_1^A, \delta_2^A, \delta_3^B, \delta_4^B)$ make $\delta_1^A \Psi_1^A + \delta_2^A \Psi_2^A \approx \delta_3^B \Psi_3^B + \delta_4^B \Psi_4^B$.

[41] The presentation of the conditions for IRP to equal their respective RTULC in sections 2.1-2.4 was conducted on the basis of market prices. Thus, the constraints $\Psi_j = \Psi_k$, $\Psi_j^A = \Psi_k^B$, or $\Psi_d^A = \Psi_d^B$ could in principle be consistent with alternative theories of competition, as has been suggested in the literature —e.g., '[t]his result can be derived either from a classical framework,

4 Stylized Facts on Profits-Wages Ratios and Measuring the Likelihood of $\Psi_j^A \approx \Psi_k^B$

4.1 Methodology and database construction

In the preceding section we demonstrated that the thesis that the long-run behavior of IRP for any pair of tradable commodities is exclusively determined by their RTULC, $\frac{p_j^A}{p_k^B} \approx \frac{\Omega_j^A}{\Omega_k^B}$, is equivalent to the hypothesis of world uniformity in TPWR, i.e., $\Psi_j^\alpha \approx \Psi$ for $j \in N$ and $\alpha = A, B$. Our reconstruction and evaluation of the theory of IRP-RC demonstrated that (i) this hypothesis has not been identified in the existing literature, (ii) there is a significant gap in knowledge regarding the statistical properties of the Ψ_j^α, and, therefore, (iii) there is currently no empirical support for the likelihood of similar total profits-wages ratios at the national and international levels, i.e., for $\Psi_j^A \approx \Psi_k^B$ and $\Psi_j^\alpha \approx \Psi^\alpha$. The same is true for the constraint of similar wage shares, $(1+\psi^A)^{-1} \approx (1+\psi^B)^{-1}$, which is a necessary constraint for the theory of IRP-RC under the hypothesis of within-country homogeneity in TPWR.

In this section we will conduct the first comprehensive empirical study of the ratios Ψ_j, $(1+\psi)^{-1}$, and $\frac{K_j}{v_j}$ and report (i) stylized facts about these ratios (Section 4.2) and (ii) empirical evidence on the hypothesis of $\Psi_j^A \approx \Psi_k^B$ (Section 4.3). For the former, we will use empirical density functions to summarize the statistical information pertaining to the ratios. For the latter, we will exploit the information obtained from the posterior distribution of the mean parameters of all countries' Ψ_j to perform a hypothesis test of $\Psi_j^A \approx \Psi_k^B$, i.e., the Bayesian estimation and comparisons of the posterior distributions of the mean of Ψ_j^α.

Appendix B describes the construction of the database. Here we provide only some details. For the study of the profits-wages ratios (ψ_j, Ψ_j) and total capital intensities $({}^{K_j}/_{v_j})$ we draw from the World Input-Output Database (WIOD) (Timmer et al., 2016). This database contains information on national input-output tables and industry-level output and input information at market prices for a period of 15 years (2000-2014) and for 43 countries, $\alpha = \text{AUS}, \text{AUT}, \ldots, \text{USA}$. This sample contains high- and middle-income countries which account for 86% of the world economy in 2016. We eliminated (i) the industries for which there is no information, leaving approximately between 33-54 industries for each country-year, and (ii) Malta as a country in our sample. Table B.1 shows the list of countries and their number of industries with nonzero information. Table B.2 describes each industry. The computation of the Ψ_j, K_j, and v_j are based on standard input-output models

along the lines of Ricardo, Sraffa, and Pasinetti, or from a mark-up pricing framework as in Eichner' (Shaikh, 1995, p. 71). Antonopoulos (1999, p. 58) shares this view. For instance, $(1+\Psi_j)$ can be interpreted as a mark-up applied to the total unit labor costs, $p_j = (1+\Psi_j)\Omega_j$. In this case $\Psi_j^A \approx \Psi_k^B$ would refer to a process of equalizing the degree of monopoly in the different industries.

relying on Leontief's inverse. The study of the wage-shares $(1+\psi)^{-1}$ will be based on the Penn World Tables, version 10 (Feenstra et al., 2015).

4.2 Total profits-wages ratios across countries and industries, Ψ_j^α

Figure 2 presents the first batch of evidence in the form of the empirical distributions of industries' TPWR within a country, $\Psi_1^\alpha, \ldots, \Psi_n^\alpha$. The first thing that stands out is the high degree of organization of the data across economies. The empirical densities show a high degree of homogeneity in their functional forms characterized by a strong clustering of the data around a single mode with limited variability characterized by slight skewness toward lower values and a fat right-hand tail. These high-value observations sometimes produce a slight bimodality, such as in Australia (AUS), Spain (ESP), and India (IND). In advanced countries, the empirical densities appear to be time-invariant while for countries undergoing a strong process of structural change, such as China (CHN) or Romania (ROU), we observe a shift in location.[42]

Remarkably, this characterization of the empirical distributions is also present in industries' TPWR *across* countries, $\Psi_j^{\text{AUS}}, \ldots, \Psi_j^{\text{USA}}$ —see Figure 3. For instance, the empirical densities for C24 refer to the total profits-wages ratios for all countries that produce manufactures of rubber and plastic products, $\Psi_{C24}^{\text{AUS}}, \ldots, \Psi_{C24}^{\text{USA}}$. Visual inspection shows that the statistical organization of the data even improves, showing more homogeneity and time invariance.[43] Distributions are smooth and unimodal, with a constrained shape with a fat right tail and a limited number of outliers. Location shifts are modest and irregularities are greatly reduced.

Under long-period positions in labor and the assumption that wages are not part of the capital advanced, we can decompose the TPWR into a distributional $\frac{r_j^\alpha}{w^\alpha}$ and a technological $\frac{K_j^\alpha}{v_j^\alpha}$ component, $\Psi_j^\alpha = \frac{r_j^\alpha}{w^\alpha}\frac{K_j^\alpha}{v_j^\alpha}$. Are the statistical regularities in the Ψ_j^α also present in the capital intensities $\frac{K_j^\alpha}{v_j^\alpha}$? If we also assume that $r_j^\alpha = r^\alpha$, then $\Psi_j^\alpha = \frac{r^\alpha}{w^\alpha}\frac{K_j^\alpha}{v_j^\alpha}$ and changes in the ratio $\frac{r^\alpha}{w^\alpha}$ should only affect the location of the within-country distribution of Ψ_j^α, leaving the rest of its statistical properties to the distribution of the $\frac{K_j^\alpha}{v_j^\alpha}$. We find evidence for this. Figure C.1 in Appendix C reports the empirical distribution of the total capital intensities $\frac{K_1^\alpha}{v_1^\alpha}, \ldots, \frac{K_n^\alpha}{v_n^\alpha}$. Visual inspection suffices to notice the strong similarity in the empirical densities of Ψ_j^α and $\frac{K_j^\alpha}{v_j^\alpha}$: the distributions are well behaved, clustering strongly around the mode with limited variability and high symmetry around the mode.[44] Thus, the statistical

[42] There are some irregular cases characterized by (i) double peak distributions, as in some years for Austria (AUT) and for all years for South Korea (KOR), and (ii) less stable functional forms, such as in Cyprus (CYP), Luxembourg (LUX), and Russia (RUS).

[43] Each density is estimated from about 42 countries. The information contained in figures 2 and 3 is exactly the same —the former is organized by country and in the latter by industry.

[44] The only exception to this being the strong bimodality of India (IND) and the slight one

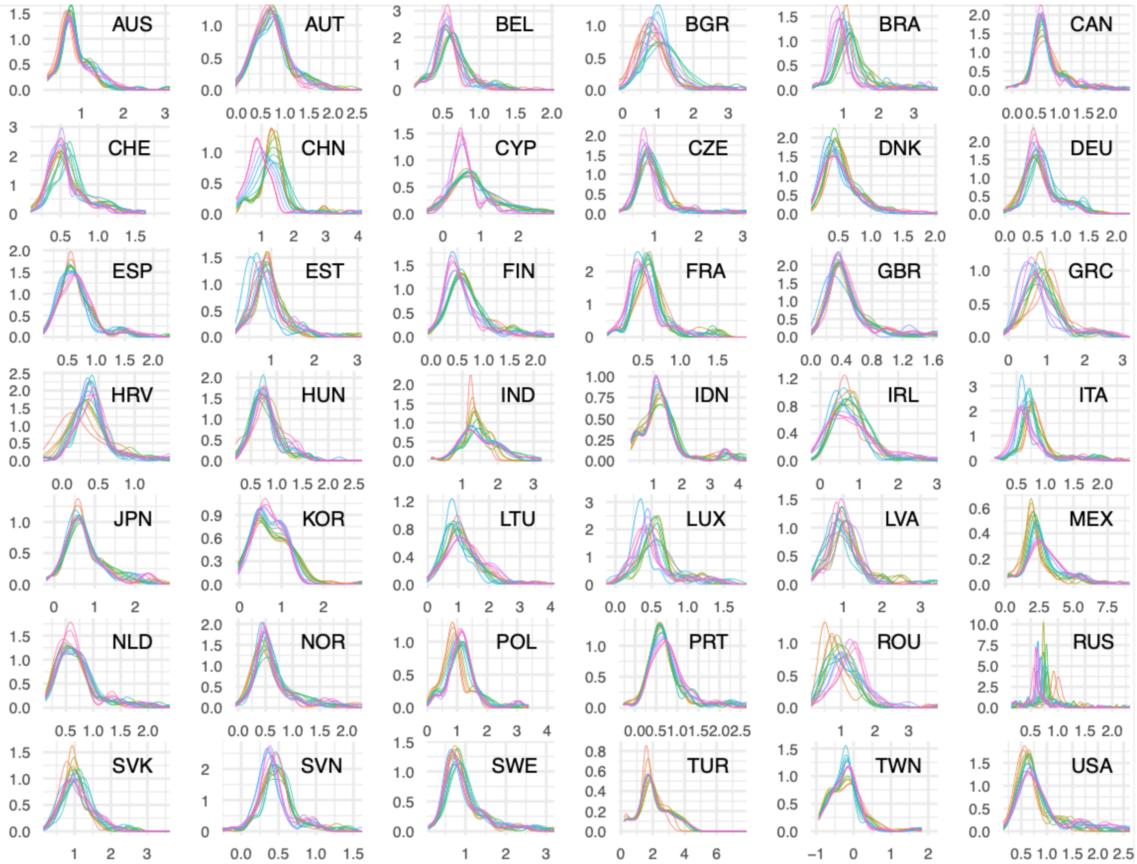

Figure 2: Total profits-wages ratios of industries within countries, 2000-2014. *Source*: authors' calculations using the World Input-Output Database.

regularities in $\frac{K_j^\alpha}{v_j^\alpha}$ seem to be relevant for Ψ_j^α.[45]

Finally, let us look at the empirical densities of the wage shares in Figure 4. The data show that $(1+\psi^\alpha)^{-1}$ share basically the same statistical properties with Ψ_j^α and $\frac{K_j^\alpha}{v_j^\alpha}$. The right panel, drawing from the WIOD, shows that the wage shares of 42 high- and middle-income countries have a highly peaked unimodal empirical density, with some changes in location and scale during the period 2000-2014. The left panel, with data for 43 countries from a wider spectrum of income levels and for the period 1970-2014, tells a similar story but with a process of structural change showing a lower average wage shares and a transition from bimodality to unimodality.

All the variates Ψ_j^α, $\frac{K_j^\alpha}{v_j^\alpha}$, and $(1+\psi^\alpha)^{-1}$ show considerable statistical organization, with persistent and generalized features such as strong concentration around a single mode and limited variability and asymmetry. This implies the existence of

in South Korea (KOR). The data from other countries, such as Russia (RUS), Greece (GRC), or China (CHN), is less organized, with strong variability in the location and the functional form.

[45] If we also consider the empirical densities of the cross country total capital intensities, $\frac{K_j^{\text{AUS}}}{v_j^{\text{AUS}}}, \frac{K_j^{\text{AUT}}}{v_j^{\text{AUT}}}, \ldots, \frac{K_j^{\text{USA}}}{v_j^{\text{USA}}}$, see Figure C.2 in Appendix C, we also observe similarities with the characteristics of the empirical densities in Figure 2, 3, and C.1.

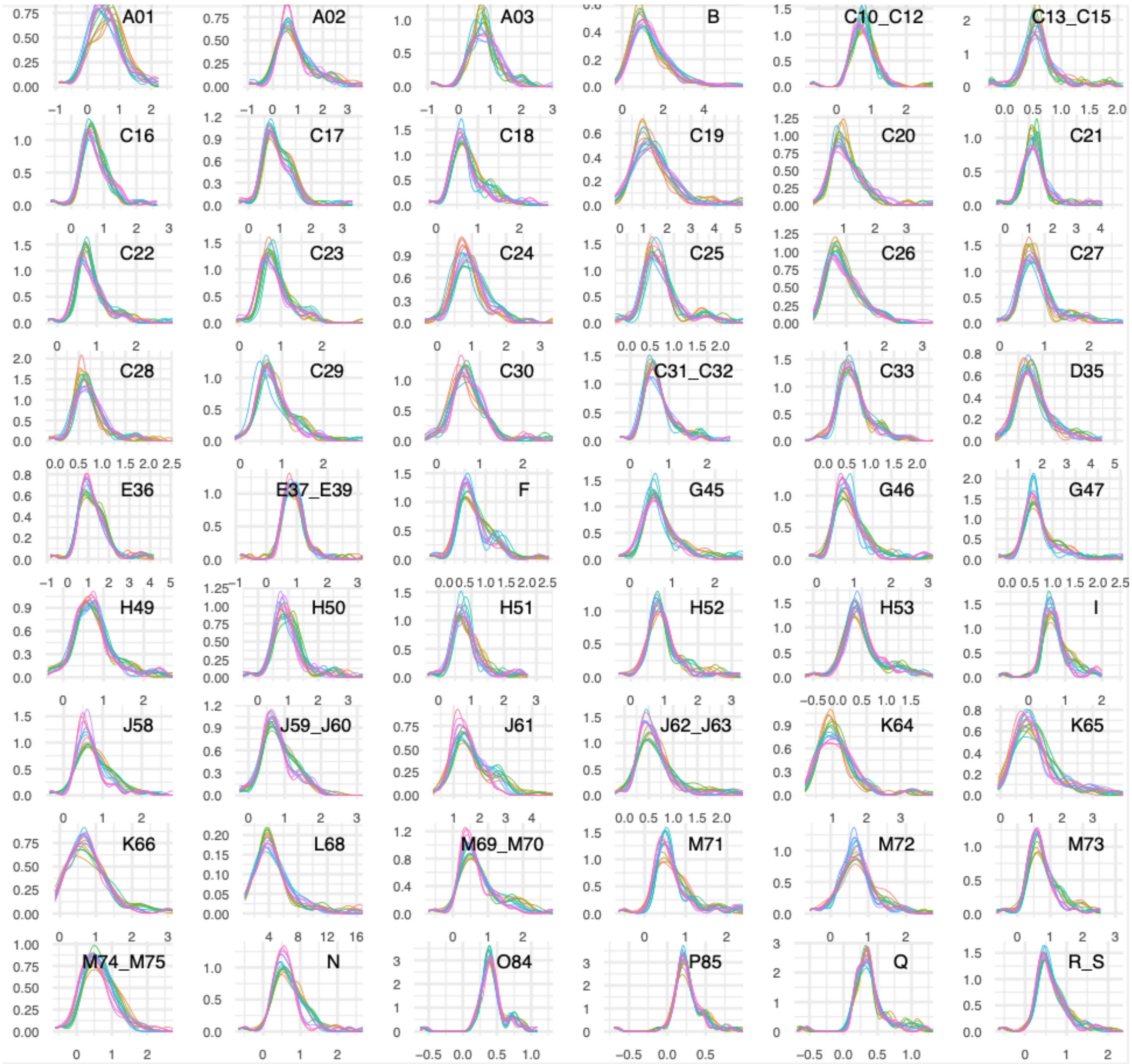

Figure 3: Total profits-wages ratios of industries across countries, 2000-2014. *Source*: authors' calculations using the World Input-Output Database.

technological and distributive constraints operating across countries and industries —Otherwise we would observe an almost uniform empirical distribution over a range of relevant values. If it is true that there is limited variability and concentration around a central value, there is also variation in location and scale. Do these statistical characteristics support the hypothesis of $\Psi_j^A \approx \Psi_k^B$? The variability of Ψ_j^α and their central tendencies may prevent the sufficiently close proximity in the TPWR required for the long-run behavior of the IRP of any pair of tradable commodity bundles to be determined exclusively by their respective RTULC.

4.3 Testing the Hypothesis of Ψ_j^A and Ψ_k^B mean-overlap

While the statistical variability of the Ψ_j^α within countries challenges the assumption that $\Psi_j^\alpha \approx \Psi^\alpha$, it is not enough to reject the assumption that $\Psi_j^A \approx \Psi_k^B$ —The

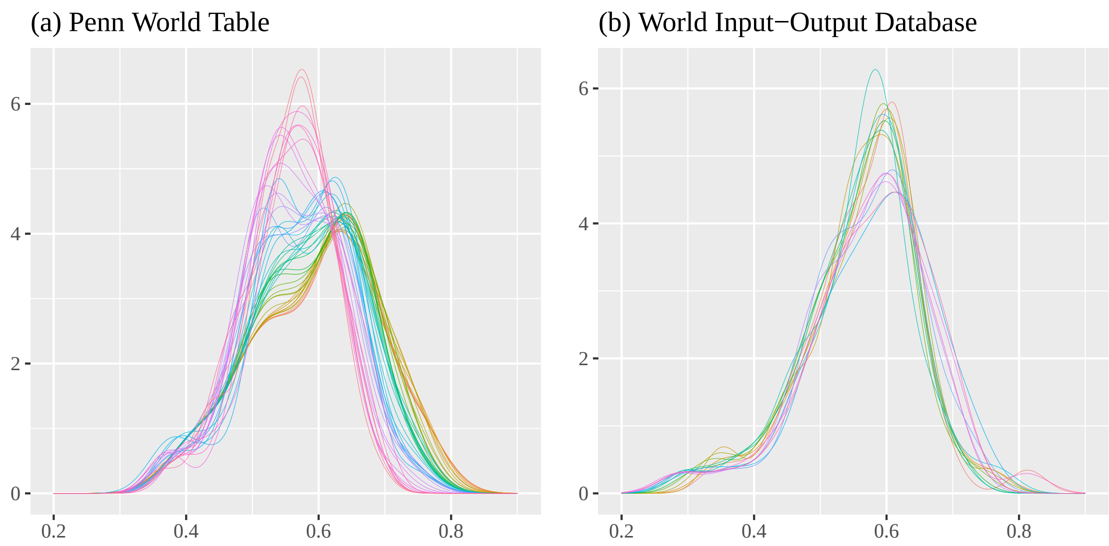

Figure 4: Wage-shares across countries: 1970-2014 for the PWT v10 and 2000-2014 for the WIOD release 2016. *Note*: Warm colors indicate older dates. *Source*: authors' calculations.

variability within countries could potentially cancel out. Given the unimodality and limited variability of the empirical distributions of the Ψ_j^α, we can take a central tendency, such as their mean μ_Ψ, to be a representative Ψ_j^α. Is there an overlap of the central tendencies of the distributions such that $\frac{1+\Psi_j^A}{1+\Psi_k^B} \approx 1$? The likelihood of the mean overlap is an approach to evaluate the hypothesis that $\Psi_j^A \approx \Psi_k^B$.

To test the empirical strength of the hypothesis, we rely on a Bayesian approach to derive the full pairwise distribution of credible values of the mean TPWR across countries —See Appendix D.1 for details on the methodology. Our approach follows Kruschke (2013), Gelman et al. (2013), and Vehtari et al. (2017) to define a hypothesis test based on the posterior distribution of the mean of Ψ_j^α. In particular, we want to know whether the means of the distributions μ_Ψ in Figure 2 are credibly different, in which case we would reject the null hypothesis that $\Psi_j^A \approx \Psi_k^B$. In this case, zero will fall outside the 95% highest density interval of the distribution of differences. To account for the credible combination of means *and* standard deviations, we compute the effect sizes of the difference of means —See Appendix D.2.

Figure 5 illustrates this approach by showing two rows of plots. The first one shows the distribution of the difference of the random draws from the posterior distribution of the mean parameter of two hypothetical countries ($\mu_{\Psi^A} - \mu_{\Psi^B}$). The second row shows the two posterior distributions from which the difference is computed ($\mu_{\Psi^A}, \mu_{\Psi^B}$). Whenever the two posterior distributions overlap, as in the plots *(a)* on the left, the mode of the resulting distribution of the differences will be located at zero. Conversely, as in the plots *(b)* on the right, almost the entire range

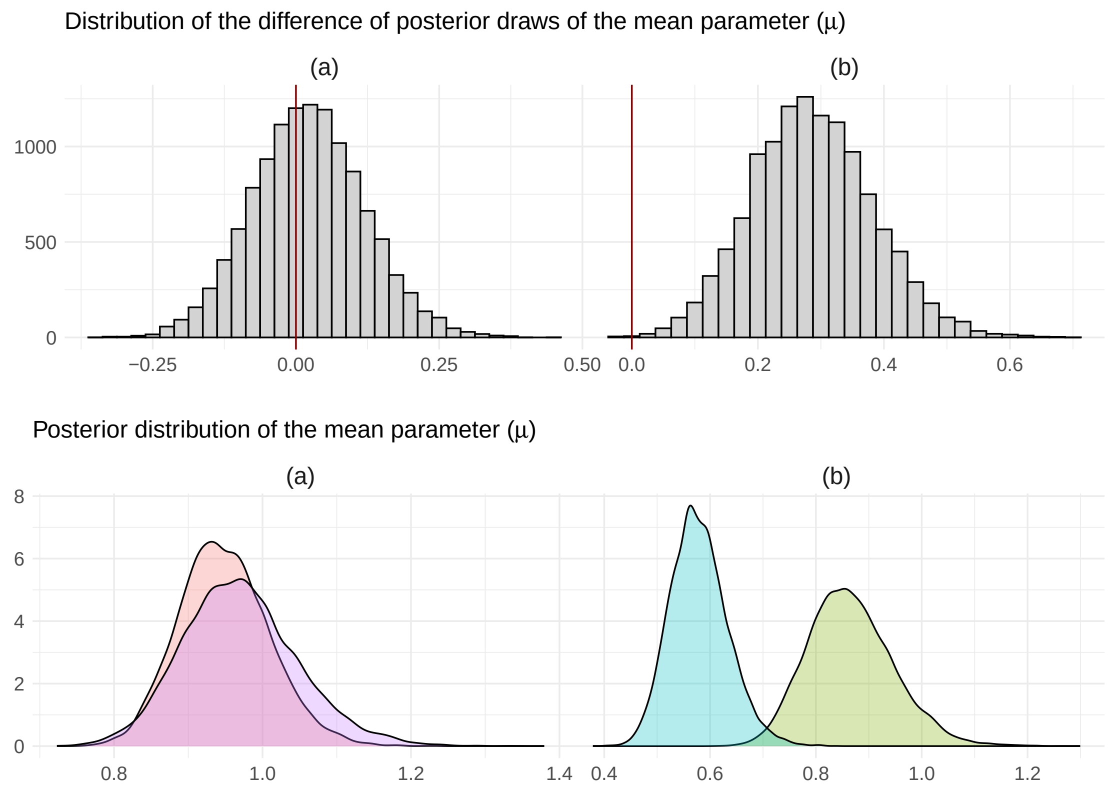

Figure 5: Illustration of the relationship between posterior distribution of the mean parameter μ_{Ψ^α} of the total profits-wage ratios Ψ_j^α and the credibility intervals of their difference. *Source*: authors' calculations.

of credible values is different from zero, indicating that the likelihood that the two original Ψ_k^α frequencies are close enough is quite low.[46] We can use this technique to evaluate the hypothesis that $\Psi_j^A \approx \Psi_k^B$ obtaining the probability that *on average* the two distributions are sufficiently close.

To derive estimates of the means of the TPWR (μ_Ψ) we must select the distribution family that best represents the data generating process behind each Ψ_j^α. Based on visual inspection of the empirical distributions of Ψ_j^α and model evaluation using leave-one-out cross-validation (Vehtari et al., 2017), we choose the Gamma distribution as a reasonable approximation of Ψ_j^α —see Appendix D.3.

Figure 6 shows the posterior distributions of the mean μ_Ψ of the Gamma model for six countries in 2014: China (CHN), Germany (DEU), France (FRA), Italy (ITA), Poland (POL), and the United States (USA). The choice of countries illustrates the variability of credible values. While the USA and China are closely aligned, Poland and Germany do not have a single point in common. Conversely,

[46] It is important to recall that Figure 5 reports the probability that the mean of the original Ψ_k^α distributions is within a certain range, not that the two distributions are bounded by the same limits.

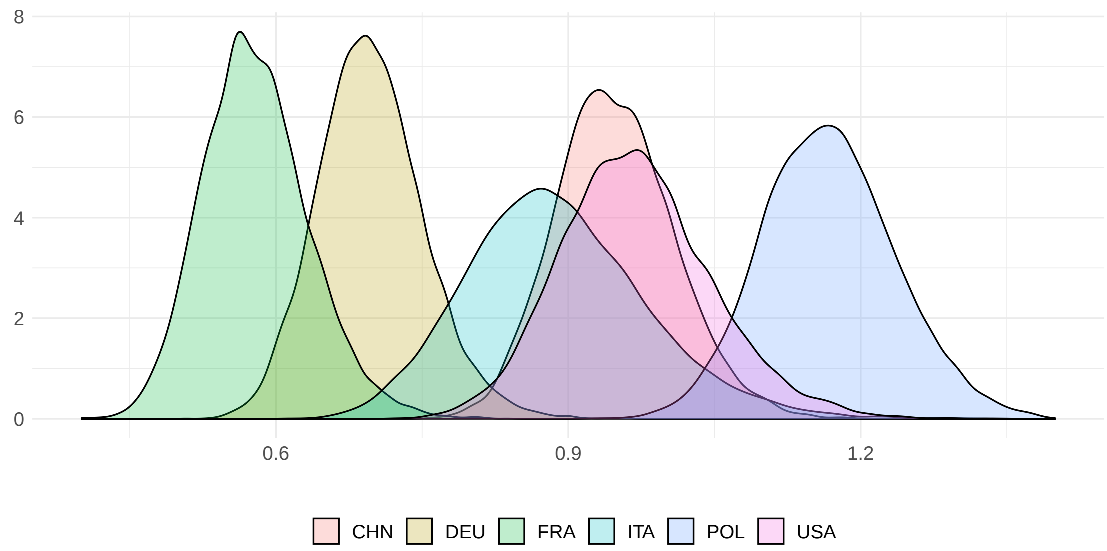

Figure 6: Posterior distribution of the mean parameter μ_{Ψ^α} of the total profits-wage ratios Ψ_j^α of a selection of countries in 2014. *Source*: authors' calculations.

France and Germany are closer, but the probability that the means of their distributions overlap is relatively low. Based on the distribution of the most likely values of the mean, we can be confident on the high likelihood that the USA and Chinese distributions are at values close enough to overlap. Conversely, the probability that μ_Ψ for France and Italy are at the same position is less than 5%. For Poland and Germany, it is highly credible that they are completely apart.

In order to generalize our test concerning $\Psi_j^A \approx \Psi_k^B$, we report in Table 1 a subset of the pairwise credibility tests for countries in 2014 —The full set of country results for 2014 can be found in Appendix D.4. The table reports only the cases where the 95% highest density interval excludes zero, i.e., the cases that would reject the null hypothesis that the average difference between the credible values of the means for the two distributions is equal to zero, $\mu_{\Psi^A} - \mu_{\Psi^B} = 0$.

The information from Table 1 challenges the assumption of $\Psi_j^A \approx \Psi_k^B$, that is, that $\frac{1+\Psi_j^A}{1+\Psi_k^B}$ is an uninformative term centered around one and, therefore, neutral in the long-run determination of IRP. First, the table shows that we can credibly reject the hypothesis that two distributions (Ψ_j^A, Ψ_k^B) have the same mean $(\mu_{\Psi^A} - \mu_{\Psi^B})$ in the majority of cases. Second, there are relevant cases where the two distributions are extremely close, such as China and the U.S., but there doesn't seem to be a strong correlation between a tight productive or trade interrelation and the proximity of the distributions. For instance, Italy and France, or Germany and China, show no overlap in the probable values of their means. This suggests that the close proximity between TPWR of some countries is neither a stylized fact nor a predictable result

	AUS	AUT	BRA	CAN	CHN	DEU	ESP	FRA	GBR	ITA	IND	JPN	KOR	MEX	POL	TUR	TWN	USA
AUS		0.03	0.36			0.1		0.28	0.26		0.56			1.81	0.16	1.17		
AUT	0.03		0.55		0.11			0.05	0.02		0.8			1.97	0.41	1.35		0.11
BEL	0.2		0.68	0.14	0.29		0.11			0.12	0.99	0.13		2.09	0.6	1.49		0.28
BGR			0.37			0.03		0.2	0.18		0.55			1.8	0.18	1.16		
BRA	0.36	0.55		0.38	0.31	0.6	0.36	0.73	0.72	0.32		0.31	0.51	0.99	0.06	0.35	0.5	0.26
CAN			0.38			0.05		0.22	0.2		0.57			1.82	0.19	1.19		
CHE	0.41	0.15	0.83	0.34	0.51	0.08	0.29			0.29	1.24	0.31	0.14	2.23	0.83	1.64		0.47
CHN		0.11	0.31			0.18		0.36	0.34		0.5		0.06	1.78	0.1	1.13	0.04	
CYP			0.46					0.04	0.02		0.65			1.86	0.28	1.24		0.01
CZE		0.17	0.29			0.24		0.42	0.4		0.51		0.11	1.78	0.09	1.13	0.08	
DEU	0.1		0.6	0.05	0.18		0.02			0.03	0.88	0.05		2.01	0.49	1.41		0.18
DNK			0.39					0.1	0.07		0.54			1.78	0.19	1.16		
ESP			0.36			0.02		0.19	0.17		0.53			1.78	0.16	1.15		
EST		0.14	0.27			0.21		0.39	0.37		0.45		0.09	1.75	0.06	1.1	0.07	
FIN	0.11		0.6	0.05	0.18		0.03			0.05	0.85	0.06		2	0.47	1.39		0.18
FRA	0.28	0.05	0.73	0.22	0.36		0.19			0.2	1.03	0.21	0.05	2.12	0.65	1.52		0.35
GBR	0.26	0.02	0.72	0.2	0.34		0.17			0.17	1.02	0.19	0.02	2.11	0.63	1.51		0.32
GRC	0.33	0.51		0.36	0.28	0.57	0.34	0.69	0.68	0.3		0.29	0.48	0.9	0.04	0.28	0.5	0.24
HRV	0.13		0.62	0.08	0.2		0.06			0.08	0.84	0.08		1.99	0.48	1.38		0.21
HUN		0.1	0.33			0.17		0.36	0.34		0.54		0.05	1.81	0.14	1.16	0.04	
IDN	0.21	0.44		0.24	0.15	0.51	0.21	0.67	0.65	0.17	0.04	0.15	0.39	1.43		0.76	0.32	0.09
IND	0.56	0.8		0.57	0.5	0.88	0.53	1.03	1.02	0.47		0.46	0.73	1.25	0.19	0.56	0.57	0.42
IRL		0.04	0.23			0.1		0.26	0.23		0.35			1.64		1.01	0.03	
ITA			0.32			0.03		0.2	0.17		0.47			1.73	0.11	1.1		
JPN			0.31			0.05		0.21	0.19		0.46			1.72	0.09	1.09		
KOR			0.51		0.06			0.05	0.02		0.73			1.92	0.35	1.3		0.07
LTU	0.2	0.43		0.23	0.15	0.5	0.21	0.66	0.64	0.16	0.04	0.15	0.38	1.42		0.77	0.33	0.09
LUX	0.27	0.04	0.72	0.21	0.34		0.18			0.19	1.02	0.21	0.04	2.11	0.64	1.51		0.33
LVA		0.14	0.3			0.22		0.39	0.37		0.51		0.09	1.78	0.09	1.14	0.06	
MEX	1.81	1.97	0.99	1.82	1.78	2.01	1.78	2.12	2.11	1.73	1.25	1.72	1.92		1.56	0.24	1.8	1.71
MLT	0.07		0.58	0.01	0.15			0.03	0.01		0.86	0.02		2.01	0.46	1.39		0.15
NLD			0.51		0.06			0.07	0.05		0.74			1.93	0.35	1.31		0.06
NOR			0.45					0.1	0.07		0.64			1.86	0.27	1.24		
POL	0.16	0.41	0.06	0.19	0.1	0.49	0.16	0.65	0.63	0.11	0.19	0.09	0.35	1.56		0.89	0.27	0.03
PRT	0.03	0.25		0.07		0.31	0.06	0.45	0.43	0.01	0.05	0.01	0.21	1.39		0.76	0.22	
ROU	0.54	0.75		0.56	0.49	0.81	0.53	0.95	0.93	0.48		0.47	0.7	1.03	0.21	0.36	0.61	0.43
RUS	0.27	0.03	0.74	0.21	0.36		0.18			0.18	1.08	0.2	0.02	2.14	0.68	1.54		0.35
SVK	0.24	0.47		0.27	0.18	0.54	0.25	0.69	0.67	0.2		0.18	0.42	1.38		0.72	0.36	0.12
SVN	0.23		0.69	0.17	0.31		0.14			0.16	0.99	0.16		2.09	0.61	1.48		0.3
SWE		0.05	0.36			0.12		0.3	0.27		0.57			1.82	0.17	1.19		
TUR	1.17	1.35	0.35	1.19	1.13	1.41	1.15	1.52	1.51	1.1	0.56	1.09	1.3	0.24	0.89		1.2	1.07
TWN			0.5		0.04						0.57			1.8	0.27	1.2		0.06
USA		0.11	0.26			0.18		0.35	0.32		0.42		0.07	1.71	0.03	1.07	0.06	

Table 1: 95% credibility bound of the pair-wise difference of posterior distributions of the mean parameters of the total profit-wage ratios for all countries ($\mu^A - \mu^B$). Missing values indicate a possible overlap with a lower than 95% credibility interval for the null. *Source*: authors' calculations.

of the similarity, proximity, or integration of different economies. Finally, in many cases the distance is quite large, as was readily apparent in Figure 2.

5 Conclusions and Discussions

This paper presents a reconstruction and evaluation of the theory of international relative prices (IRP) based on the theory of "real competition" (RC), i.e., the theory of IRP-RC, developed by Anwar Shaikh and applied to a diverse range of economies. The main thesis of the theory is that the long-run behavior of IRP for any pair of tradable commodity bundles is exclusively determined by their relative total unit labor costs (RTULC). This thesis is equivalent to the constraint that the total profits-wages ratios (TPWR) of these two commodity bundles are sufficiently similar. To sustain this result, the literature has put forth a set of hypotheses ($\mathcal{H}_1$ and $\mathcal{H}_2$) and arguments ($\mathcal{A}_1 - \mathcal{A}_5$) which, according to their proponents, are rooted in the classical theory of competition.

The theoretical and empirical results presented in this paper raise questions concerning the soundness of the theory of IRP-RC. From an accounting perspective, the proposed hypotheses are *inefficient* to produce the desired thesis. In particular, Proposition 2 demonstrated that the assertion that the terms of trade of any pair of tradable commodities are equal their RTULC requires world uniformity in TPWR, $\Psi_j^A = \Psi_k^B = \Psi$, rather than the uniformity *within a single country* $\Psi_j^A = \Psi^A$ and $\Psi_k^B = \Psi^B$ that is postulated in the theory. Not only has this condition not been identified in the existing literature but it is also inconsistent with the purported significant cross-country differences in technology and distribution, as highlighted in their critique of the neoclassical theory of international trade.

On the other hand, the theoretical and empirical arguments employed to constrain the TPWR are *weak* due to their speculative nature. Our analysis revealed that none of the arguments presented in $\mathcal{A}_1$ to $\mathcal{A}_5$ nor the forces of competition are sufficient to constrain the TPWR in a manner that would yield the proposed nor the necessary hypotheses to derive the desired thesis.

We showed that the knowledge of the literature on the statistical properties of TPWR in actual economies is scarce. In light of this dearth of knowledge, we undertook the first comprehensive empirical investigation of the TPWR, capital intensities, and wage shares, drawing upon the WIOD and PWT databases. The functional form of the empirical densities of these variates *across industries within countries* $\Psi_1^\alpha, \ldots, \Psi_n^\alpha$ and $\frac{K_1^\alpha}{v_1^\alpha}, \ldots, \frac{K_n^\alpha}{v_n^\alpha}$ exhibited high homogeneity characterized by a strong clustering of the data around a single mode with limited variability characterized by a slight skewness towards lower values and a fat right-hand tail. The same general patterns were identified for the variates *across countries within industries* $\Psi_j^{\text{AUS}}, \ldots, \Psi_j^{\text{USA}}$ and $\frac{K_j^{\text{AUS}}}{v_j^{\text{AUS}}}, \ldots, \frac{K_j^{\text{USA}}}{v_j^{\text{USA}}}$. We employed Bayesian methods to study the posterior probability distribution of the means of these TPWR and discovered that, despite the aforementioned stylized facts, with the exception of a few select pairs of countries, such as China and the United States, pairs of (Ψ_j^A, Ψ_k^B) exhibit notable discrepancies, raising questions about the long-run neutrality of the TPWR in the determination of IRP in general.

How our theoretical and empirical results relate to (i) the empirical evidence on the long-run association between IRP and RTULC from the literature and (ii) the theory of IRP based on the principle of absolute advantage under long-period positions?

In regard to (i), the only evidence that can be considered consistent with the theory of IRP-RC is the empirical evidence reported in the literature on the long-run association between IRP and their respective RTULC. However, *these regularities cannot be explained by the theory*, thus representing a conundrum within the existing literature. Now, our paper does not evaluate this empirical association between IRP and RTULC. However, the lack of sound economic arguments and limited empirical evidence to sustain $\Psi_j^A \approx \Psi_k^B$ suggests that this literature requires revision. Furthermore, the existence of a long-run association between IRP and RTULC does not imply that the techno-distributive information embedded in Ψ_j^α is neutral. The literature adopts the hypothesis of inter-country homogeneity in TPWR, i.e., $\Psi_j^A = \Psi^A$ and $\Psi_k^B = \Psi^B$. Consequently, under this hypothesis, Proposition 3 and 5 indicate that $\frac{1+\psi^A}{1+\psi^B}$ should be included as a regressor in the statistical exercises.[47]

As for (ii), we want to examine two implications. First, in the absence of any reason to dismiss the techno-distributive elements inherent in the relative TPWR, it is necessary to consider *full* production prices in the study of the long-run behavior of IRP, rather than solely the RTULC. To this end, the study of the causes and effects of IRP's levels and dynamics can be conducted using open-economy Sraffian models.[48] Under this theoretical framework, it would be pertinent to evaluate the role played by the commodity bundles (d^A, d^B) in the determination of IRP, a dimension that has been overlooked in the literature on the theory of IRP-R. For instance, consider the reported variability in the Ψ_j^α across countries. It might be that the proportions of commodity baskets (d^A, d^B) are the factors producing the empirical association between IRP indices and their RTULC.

Second, the persistent and generalized statistical characteristics of the Ψ_j and $\frac{K_j}{v_j}$ contribute to the regularities observed in other elements of the productive and allocative structure in empirical linear production models.[49] If the empirical regu-

[47] Assuming with this that the relative $\sum_{i=1}^{n} \sigma_i^\alpha m_{ij}^\alpha$ in (15) and (18) are uninformative components. Fevereiro (2019) studies the relationship between the wage-share and the undervaluation of the RER. Martínez-González et al. (2019) present evidence supporting the hypothesis that the rates of surplus value in developed countries are lower than those in underdeveloped countries. This evidence suggests that the term $\frac{1+\psi^A}{1+\psi^B}$ may be a relevant factor in determining IRP.

[48] See Parrinello (2009), Vasudevan (2012), Bellino and Fratini (2022).

[49] Mariolis and Tsoulfidis (2011) and Shaikh et al. (2023) report robust statistical regularities

larities observed in Ψ_j and $\frac{K_j}{v_j}$ are indeed inconsistent with the hypothesis that is necessary to sustain the main thesis of the theory of IRP-RC, it also implies that the uninformative structure of the techniques of production and social output that are implicit in typical linear production models are rejected. These models are sufficiently general to demonstrate the existence of economically meaningful prices and quantities; however, they are unable to account for their empirical regularities. The statistical structure of the Ψ_j and $\frac{K_j}{v_j}$ indicates the presence of pertinent technological and distributive constraints. It is necessary to identify a compromise position, as has been achieved in the literature on the explanation of empirical regularities in price- and wage-profits curves.[50]

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on the eigenvalues of the input-coefficient matrices and Torres-González and Yang (2019) for their column and row sums as well as in their left and right Perron-Frobenius (P-F) eigenvectors. Torres-González (2022) and Ferrer-Hernández and Torres-González (2022) report a statistical tendency towards the proportionality between the labor vector and the left P-F eigenvector. Torres-González (2022) finds results consistent to those in Section 4 for the U.S. economy in 1977-2012 (between 402-530 industries).

[50] See Mariolis and Tsoulfidis (2011), Schefold (2013, 2023), and Ferrer-Hernández and Torres-González 2022.

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A The construction of the total profits-wages ratios Ψ_j and their alternative representations

A.1 Obtaining Equation (3)

Let x_j be the value of output of industry j. From a cost accounting perspective, market value x_j can be decomposed into

$$x_j = \text{wages}_j + \text{profits}_j + \text{value of means of production}_j. \tag{A.1}$$

Value added is represented by $\text{wages}_j + \text{profits}_j$, while intermediate consumption by the value of means of production_j. Under the assumptions of the economy given in 2.1, the value of output of any industry can be expressed as $x_j = p_j q_j$, where p_j is the market price of commodity j and q_j the quantity of this commodity produced. Therefore,

$$p_j = \omega_j + \pi_j + \lambda_j \quad \text{for} \quad j \in N, \tag{A.2}$$

where $\omega_j \equiv \frac{\text{wages}_j}{q_j} > 0$,[A.1] $\pi_j \equiv \frac{\text{profits}_j}{q_j} \geqslant 0$, and $\lambda_j \equiv \frac{\text{value of means of prod.}_j}{q_j} \geqslant 0$ represent the labor costs, the profits, and the costs of the means of production in industry j, all of them per unit of output j. These inequalities imply that value added in each industry is positive.

Now, the unit value of the means of production λ_j can be further decomposed into the wages $\omega_j^{(1)}$, the profits $\pi_j^{(1)}$, and value of the means of production $\lambda_j^{(1)}$ incurred by *all the suppliers to industry j necessary to produce one unit of commodity j*,

$$\lambda_j = \omega_j^{(1)} + \pi_j^{(1)} + \lambda_j^{(1)} \quad j \in N. \tag{A.3}$$

But this process can now be repeated for the suppliers of the suppliers of commodity j, and so on and so forth, so that for $j \in N$ we have

$$\lambda_j^{(1)} = \omega_j^{(2)} + \pi_j^{(2)} + \lambda_j^{(2)} \tag{A.4}$$

$$\lambda_j^{(2)} = \omega_j^{(3)} + \pi_j^{(3)} + \lambda_j^{(3)} \tag{A.5}$$

$$\vdots \tag{A.6}$$

$$\lambda_j = \omega_j^{(1)} + \pi_j^{(1)} + \omega_j^{(2)} + \pi_j^{(2)} + \omega_j^{(3)} + \pi_j^{(3)} + \cdots + \omega_j^{(\tau)} + \pi_j^{(\tau)} + \lambda_j^{(\tau)} \tag{A.7}$$

$$= \sum_{\nu=1}^{\tau-1} (\omega_j^{(\nu)} + \pi_j^{(\nu)}) + (\omega_j^{(\tau)} + \pi_j^{(\tau)} + \lambda_j^{(\tau)}) \tag{A.8}$$

Let $\{\omega_j^{(\nu)}\}$, $\{\pi_j^{(\nu)}\}$, and $\{\lambda_j^{(\nu)}\}$ be a sequence for $\nu = 0, 1, 2, \ldots$, each ν representing a specific segment of the value chain of commodity j. Let $\omega_j^{(0)} \equiv \omega_j$, $\pi_j^{(0)} \equiv \pi_j$, and $\lambda_j^{(0)} \equiv \lambda_j$. Because value added is positive for each ν, that is, $\omega_j^{(\nu)} + \pi_j^{(\nu)} = \lambda_j^{(\nu)} -$

[A.1] Labor is indispensable.

$\lambda_j^{(\nu+1)} > 0$, this means that the sequence $\{\lambda_j^{(\nu)}\}$ is strictly decreasing, $\lambda_j^{(\nu)} > \lambda_j^{(\nu+1)}$. In addition, the sequence $\{\lambda_j^{(\nu)}\}$ is bounded from bellow, $\lambda_j^{(\nu)} \geqslant 0$. This means that the sequence $\{\lambda_j^{(\nu)}\} = \{\omega_j^{(\nu+1)} + \pi_j^{(\nu+1)} + \lambda_j^{(\nu+1)}\}$ converges to zero, that is, $\lim_{\nu\to\infty} \lambda_j^{(\nu)} = \omega_j^{(\nu+1)} + \pi_j^{(\nu+1)} + \lambda_j^{(\nu+1)} = 0$, and therefore $\sum_{k=1}^{\infty}(\omega_j^{(\nu)} + \pi_j^{(\nu)}) < \infty$.[A.2] Hence, for $j \in N$ we have

$$\lambda_j = \sum_{\nu=1}^{\infty} \omega_j^{(\nu)} + \sum_{\nu=1}^{\infty} \pi_j^{(\nu)} = \omega_j^I + \pi_j^I, \tag{A.9}$$

where $\omega_j^I \equiv \sum_{\nu=1}^{\infty} \omega_j^{(\nu)}$ and $\pi_j^I \equiv \sum_{\nu=1}^{\infty} \pi_j^{(\nu)}$. Substituting (A.9) into (2) we get (3):

$$p_j = (\omega_j + \omega_j^I) + (\pi_j + \pi_j^I) = \Omega_j + \Pi_j \tag{A.10}$$

where $\Omega_j \equiv \omega_j + \omega_j^I = \sum_{\nu=0}^{\infty} \omega_j^{(\nu)}$ is the *total*, or direct plus indirect, unit labor costs and $\Pi_j \equiv \pi_j + \pi_j^I = \sum_{\nu=0}^{\infty} \pi_j^{(\nu)}$ is the total unit profits.

A.2 Shaikh's representation of Ψ_j

Shaikh (1984, p. 68) provides the decomposition of total profits-wages ratios Ψ_j as an infinite convex combination of *average* direct profit-wages ratios in each stage of the value chain of a commodity $\psi_j^{(\nu)}$ with weights given by the shares of the direct labor in stage ν in total labor $\frac{l_j^{(\nu)}}{v_j}$, i.e.,

$$\Psi_j = \psi_j \frac{l_j}{v_j} + \psi_j^{(1)} \frac{l_j^{(1)}}{v_j} + \psi_j^{(2)} \frac{l_j^{(2)}}{v_j} + \cdots, \tag{A.11}$$

where $\psi_j^{(\nu)} \equiv \frac{\pi_j^{(\nu)}}{\omega_j^{(\nu)}}$, $l_j^{(\nu)} \equiv \frac{L_j^{(\nu)}}{q_j}$, $L_j^{(\nu)}$ is the quantity of labor necessary in the production of means of production in stage ν in order to produce q_j, $v_j \equiv \frac{V_j}{q_j}$, and $V_j = \sum_{\nu=0}^{\infty} L_j^{(\nu)}$ is the total quantity of labor necessary in the production q_j.

To obtain (A.11) we first define the total profits-wages ratios:

$$\Psi_j \equiv \frac{\Pi_j}{\Omega_j}. \tag{A.12}$$

Second, assume long-period positions for labor.[A.3] Hence, we can express wages per unit of output of industry j in any stage of the value chain as $\omega_j^{(\nu)} = \frac{wL_j^{(\nu)}}{q_j}$ for $\nu = 0, 1, 2, \ldots$. Previously we defined $l_j^{(\nu)} \equiv \frac{L_j^{(\nu)}}{q_j}$ and $v_j \equiv \frac{V_j}{q_j}$. Therefore,

$$\omega_j^{(\nu)} = wl_j^{(\nu)} \tag{A.13}$$

$$\Omega_j = \sum_{\nu=0}^{\infty} \omega_j^{(\nu)} = w \sum_{\nu=0}^{\infty} l_j^{(\nu)} = wv_j \tag{A.14}$$

[A.2] See Nikaido (1968, Theorem 15.2, p. 112).
[A.3] See Section 2.5.

Finally, given (A.12), (A.14), $\Pi_j = \sum_{\nu=0}^{\infty} \pi_j^{(\nu)}$, and $\psi_j^{(\nu)} \equiv \frac{\pi_j^{(\nu)}}{\omega_j^{(\nu)}} = \frac{\pi_j^{(\nu)}}{wl_j^{(\nu)}}$ we have

$$\Psi_j = \frac{\pi_j + \pi_j^{(1)} + \pi_j^{(2)} + \cdots}{wv_j} \tag{A.15}$$

$$= \frac{\pi_j}{wv_j} + \frac{\pi_j^{(1)}}{wv_j} + \frac{\pi_j^{(2)}}{wv_j} + \cdots \tag{A.16}$$

$$= \frac{\pi_j}{wv_j}\frac{wl_j}{wl_j} + \frac{\pi_j^{(1)}}{wv_j}\frac{wl_j^{(1)}}{wl_j^{(1)}} + \frac{\pi_j^{(2)}}{wv_j}\frac{wl_j^{(2)}}{wl_j^{(2)}} + \cdots \tag{A.17}$$

$$= \frac{\pi_j}{wl_j}\frac{l_j}{v_j} + \frac{\pi_j^{(1)}}{wl_j^{(1)}}\frac{l_j^{(1)}}{v_j} + \frac{\pi_j^{(2)}}{wl_j^{(2)}}\frac{l_j^{(2)}}{v_j} + \cdots \tag{A.18}$$

$$= \psi_j \frac{l_j}{v_j} + \psi_j^{(1)}\frac{l_j^{(1)}}{v_j} + \psi_j^{(2)}\frac{wL_j^{(2)}}{wL_j^{(2)}} + \cdots \quad . \tag{A.19}$$

The term $\psi^{(\nu)} \equiv \frac{\pi_j^{(\nu)}}{wl_j^{(\nu)}}$ is the ratio of profits and wages of the industries participating in stage ν of the value chain, and $\frac{l_j^{(\nu)}}{v_j}$ is the share of the indirect labor contained in stage ν of the value chain in the total labor contained in the production of commodity j. Note that each $\psi^{(\nu)}$ is a weighted average of the direct profits-wages ratios only of the industries participating in the stage ν of the value chain. Since the value chain is potentially composed of an infinite number of stages, this representation of Ψ_j (1) requires an infinite number of weighted averages of the direct profits-wages and (2) aggregates the direct profits-wages ratios of different industries, so it is not possible to identify the specific contribution of each industry.

A.3 Obtaining equation (5) from a linear production model. The total profits-wages ratios Ψ_j as a convex combination of the direct profits-wages Ratios ψ_j

Our proposed decomposition of total profit-wages ratios $\Psi_j = \frac{\Pi_j}{\Omega_j}$ in (4) is

$$\Psi_j = \frac{\Pi_j}{\Omega_j} = \psi_1 m_{1j} + \psi_2 m_{2j} + \cdots + \psi_n m_{nj} \quad \text{for} \quad j \in N. \tag{A.20}$$

Let us now derive the weights $m_{1j}, m_{2j}, \ldots, m_{nj}$ and obtain (A.20) using a linear production model. Specifically, we use Pasinetti (1973, Section 2) production model where fixed capital is treated as a durable means of production with uniform exponential depreciation rates for each commodity-input within each industry.

Let us start with (A.1) and $x_j = p_j q_j$. The value of the means of production of commodities $j \in N$ can be decomposed as price × quantity magnitudes:

$$\text{value of means of production}_j = \sum_{i=1}^{n} p_i Q_{ij} + p_i D_{ij} = \sum_{i=1}^{n} p_i (Q_{ij} + D_{ij}), \tag{A.21}$$

where p_j is the market price of the j-th commodity, q_j is the quantity of commodity j produced, Q_{ij} is the quantity of commodity i used as an input in the production of commodity j, and D_{ij} is the quantity of durable commodity i depreciated in industry j. Note that (i) industries do not use imported means of production, (ii) there are no assumptions about the input-output relationships, and (iii) there are no constraints on the labor inputs and the composition of the capital advanced as well as in their rates of return.

Now, consider the following options: (a) the quantities q_j, Q_{ij}, and D_{ij}, for $i, j \in N$, are given and *define* the ratios $a_{ij} \equiv \frac{Q_{ij}}{q_j}$ and $d_{ij} \equiv \frac{D_{ij}}{q_j}$ for $i, j \in N$, respectively; or (b) there is proportionality between commodity means of production and depreciation and output, $Q_{ij} = a_{ij}q_j$ and $D_{ij} = d_{ij}q_j$ for $i, j \in N$, respectively. Assuming one of the two options we can express (2) as

$$\begin{aligned} p_j &= \sum\nolimits_{i=1}^{n} p_i \frac{Q_{ij} + D_{ij}}{q_j} + \frac{\text{wages}_j}{q_j} + \frac{\text{profits}_j}{q_j} \\ &= \sum\nolimits_{i=1}^{n} p_i a_{ij} + \sum\nolimits_{i=1}^{n} p_i d_{ij} + \omega_j + \pi_j \quad \text{for} \quad j \in N. \end{aligned} \tag{A.22}$$

Expressing (A.22) in matrix notation we have:

$$\begin{aligned} \mathbf{p}' &= \mathbf{p}'(\mathbf{A} + \mathbf{D}) + \boldsymbol{\omega}' + \boldsymbol{\pi}' = \mathbf{p}'\mathbf{A}^{\ominus} + \boldsymbol{\omega}' + \boldsymbol{\pi}' \\ \mathbf{p}'\left(\mathbf{I} - \mathbf{A}^{\ominus}\right) &= \boldsymbol{\omega}' + \boldsymbol{\pi}', \end{aligned} \tag{A.23}$$

where $\underset{(1\times n)}{\mathbf{p}'} \equiv [p_j]$ is the price vector, $\underset{(n\times n)}{\mathbf{A}} \equiv [a_{ij}] \equiv [\frac{Q_{ij}}{q_j}] \geqq \mathbf{0}$ is the physical input-coefficients matrix, $\underset{(n\times n)}{\mathbf{D}} \equiv [d_{ij}] \equiv [\frac{D_{ij}}{q_j}] \geqq \mathbf{0}$ is the physical depreciation-coefficients matrix, $\mathbf{A}^{\ominus} \equiv \mathbf{A} + \mathbf{D}$, $\underset{(1\times n)}{\boldsymbol{\omega}'} \equiv [\omega_j]$ is the vector with the direct wages per unit of output, and $\underset{(1\times n)}{\boldsymbol{\pi}'} \equiv [\pi_j]$ is the vector with the direct profits per unit of output.

Assume that $\mathbf{A}^{\ominus}$ is viable so $(\mathbf{I} - \mathbf{A}^{\ominus})$ is non-singular and $(\mathbf{I} - \mathbf{A}^{\ominus})^{-1} = \mathbf{I} + \sum_{k=1}^{\infty}(\mathbf{A}^{\ominus})^k \geqslant \mathbf{0}$ is semipositive with a positive diagonal.[A.4] Matrix $\boldsymbol{\Lambda} \equiv (\mathbf{I} - \mathbf{A}^{\ominus})^{-1} \equiv [\Lambda_{ij}]$ is called Leontief's inverse. Then, solving (A.23) for $\mathbf{p}$ we get

$$\mathbf{p}' = \boldsymbol{\omega}'\boldsymbol{\Lambda} + \boldsymbol{\pi}'\boldsymbol{\Lambda} = \boldsymbol{\Omega}' + \boldsymbol{\Pi}', \tag{A.24}$$

where $\boldsymbol{\omega}'\boldsymbol{\Lambda} \equiv \boldsymbol{\Omega}' \equiv [\Omega_j]$ is the vector with the total wages per unit of output, and $\boldsymbol{\pi}'\boldsymbol{\Lambda} \equiv \boldsymbol{\Pi}' \equiv [\Pi_j]$ is the vector with the total profits per unit of output.

Let any row vector with 'hat' be a squared diagonal matrix with the elements of that vector in the main diagonal. Hence $\hat{\boldsymbol{\Omega}} = \text{diag}\{\Omega_1, \ldots, \Omega_n\}$ and $\hat{\boldsymbol{\Omega}}^{-1} = \text{diag}\{\frac{1}{\Omega_1}, \ldots, \frac{1}{\Omega_n}\}$. Note that $\boldsymbol{\omega}' = \mathbf{1}'\hat{\boldsymbol{\omega}}$, where $\mathbf{1}' \equiv [1]$ is the summation vector —a vector with ones. With these definitions, and remembering that $\boldsymbol{\Omega}' = \boldsymbol{\omega}'\boldsymbol{\Lambda}$, we can obtain the matrix with the weights m_{ij} for $i, j \in N$ which will be used in the

[A.4] If $\mathbf{A}^{\ominus}$ is in addition irreducible, then $(\mathbf{I} - \mathbf{A}^{\ominus})^{-1} > \mathbf{0}$ (Takayama, 1974, p. 392).

construction of the total profits-wages ratios:

$$\mathbf{1}' = \mathbf{\Omega}'\hat{\mathbf{\Omega}}^{-1} = \boldsymbol{\omega}'\mathbf{\Lambda}\hat{\mathbf{\Omega}}^{-1} = \mathbf{1}'\hat{\boldsymbol{\omega}}\mathbf{\Lambda}\hat{\mathbf{\Omega}}^{-1} = \mathbf{1}'\mathbf{M}, \tag{A.25}$$

where $\hat{\boldsymbol{\omega}}\mathbf{\Lambda}\hat{\mathbf{\Omega}}^{-1} \equiv \mathbf{M} \equiv [m_{ij}] \geqslant \mathbf{0}$ is the matrix with weights in (5), the column sums of which are equal to one. Note that $m_{ij} = \frac{\omega_i \Lambda_{ij}}{\Omega_j}$. Since

$$\mathbf{1}'\mathbf{M} = \mathbf{1}', \tag{A.26}$$

then $\mathbf{M}$ is a column stochastic matrix with a dominant eigenvalue $\rho(\mathbf{M}) = 1$ associated with the positive eigenvector $\mathbf{1}'$. Thus, the columns of the matrix $\mathbf{M}$ tell us the industry location of the indirect or direct unit labor costs corresponding to the total unit labor costs in industry j.

Under long-period positions labor is homogeneous and each unit of simple labor receives a uniform wage rate. Therefore, we can aggregate the labor in each industry L_j, obtain $l_j = \frac{L_j}{q_j}$ as in (a) and (b) above, and express $\omega_j = \frac{\text{wages}_j}{q_j} = \frac{wL_j}{q_j} = wl_j$. In addition, the wage rate in the numerator and denominator of each $m_{ij} = \frac{\omega_i \Lambda_{ij}}{\Omega_j} = \frac{wl_j \Lambda_{ij}}{wv_j}$ is canceled and $\mathbf{M} = \hat{\mathbf{l}}\mathbf{\Lambda}\hat{\mathbf{v}}^{-1} = [\frac{l_i \Lambda_{ij}}{v_j}]$ and $v_j = \sum_{i=1}^{n} l_i \Lambda_{ij}$.

Now, we can express (A.24) as in (3):

$$\mathbf{p}' = \mathbf{\Omega}' + \mathbf{\Pi}' = \mathbf{\Omega}'(\mathbf{I} + \hat{\mathbf{\Pi}}\hat{\mathbf{\Omega}}^{-1}) = \mathbf{\Omega}'(\mathbf{I} + \hat{\mathbf{\Psi}}), \tag{A.27}$$

where $\hat{\mathbf{\Pi}}\hat{\mathbf{\Omega}}^{-1} \equiv \hat{\mathbf{\Psi}} \equiv \text{diag}\{\frac{\Pi_1}{\Omega_1}, \ldots, \frac{\Pi_n}{\Omega_n}\}$ is the diagonal matrix with the total profits-wages ratios. Define the vector with the *direct* profits-wages ratios as $\boldsymbol{\pi}'\hat{\boldsymbol{\omega}}^{-1} \equiv \boldsymbol{\psi}' \equiv [\psi_j] \equiv [\frac{\pi_j}{\omega_j}]$. This means that $\boldsymbol{\pi}' = \boldsymbol{\psi}'\hat{\boldsymbol{\omega}}$. Remembering that $\mathbf{\Pi}' \equiv \boldsymbol{\pi}'\mathbf{\Lambda}$, then

$$\mathbf{\Psi}' \equiv \mathbf{1}'\hat{\mathbf{\Psi}} = \mathbf{1}'\hat{\mathbf{\Pi}}\hat{\mathbf{\Omega}}^{-1} = \mathbf{\Pi}'\hat{\mathbf{\Omega}}^{-1} = \boldsymbol{\pi}'\mathbf{\Lambda}\hat{\mathbf{\Omega}}^{-1} = \boldsymbol{\psi}'\hat{\boldsymbol{\omega}}\mathbf{\Lambda}\hat{\mathbf{\Omega}}^{-1} = \boldsymbol{\psi}'\mathbf{M}, \tag{A.28}$$

$$\Psi_j = \boldsymbol{\psi}'\mathbf{M}_{(j)} = \sum_{i=1}^{n} \psi_i m_{ij} \quad \text{for} \quad j \in N, \tag{A.29}$$

so Ψ_j is a convex linear combination, or a weighted average, with the $\{m_{1j}, \ldots, m_{nj}\}$ as weights, consistent with (4).

Note that although the same n ratios ψ_i go into in each Ψ_j, the weights m_{ij} into which the ψ_i participates in each Ψ_j may be different in each industry. Therefore, $\Psi_j = \Psi_k$ for $j, k \in N$ requires n weighting systems such that the n weighted averages of the same direct profits-wages ratios $\sum_{i=1}^{n} \psi_i m_{ij}$ are all equal. On the other hand, if all the *direct* profits-wages ratios are equal $\psi_j = \psi$, then $\Psi_j = \psi \sum_{i=1}^{n} m_{ij} = \psi$ for any weighting system. Thus, small variability in the direct profits-wages ratios ψ_j coupled with a set of weights that maintains or reduces this variability can contribute to maintain small variability in the total profits-wages ratios Ψ_j.

We identify the following advantages of our decomposition (A.20) vis-a-vis the one proposed by Shaikh (1984) (A.11). Firstly, each Ψ_j depends on the same n

direct ratios ψ_j, so the contributions of each ψ_j can be identified. Second, given that $m_{ij} = \frac{l_i \Lambda_{ij}}{v_j}$, the weights $(m_{1j}, m_{2j}, \ldots, m_{nj})$ for each Ψ_j represent the share of the total labor conducted in industry i, $l_i \Lambda_{ij}$, in the total labor of the society in order to produce one unit of net output of commodity j, $v_j = \sum_{i=1}^{n} l_i \Lambda_{ij}$. Third, there is a finite number of ψ_j and $(m_{1j}, m_{2j}, \ldots, m_{nj})$ for each Ψ_j.

A.4 Some statistical properties of Ψ_j

We now show that Ψ_j depends on the statistical characteristics of *all* the direct ratios ψ_j, the weights m_{ij} for $i \in N$, and their relation. Given that for any two random variables $\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$ and that $\bar{m}_j \equiv \frac{1}{n} \sum_{i=1}^{n} m_{ij} = \frac{1}{n}$ for $j \in N$, then we have:

$$\Psi_j \equiv \sum_{i=1}^{n} \psi_i m_{ij} = n\text{Cov}(\psi_i, m_{ij}) + \bar{\psi} \quad \text{for} \quad j \in N, \tag{A.30}$$

where $\bar{\psi} \equiv \frac{1}{n} \sum_{i=1}^{n} \psi_i$ is a simple average of all industries and does not necessarily represent the economy-wide profits-wages ratio ψ in (7). Given that $\bar{\psi}$ in (A.30) is the same for each Ψ_j, the variability of the Ψ_j depends then on the covariance between the direct ψ_i and the weights m_{ij}. But the same n ratios ψ_i participate in each Ψ_j, so the more similar the n columns $(m_{1j}, m_{2j}, \ldots, m_{nj})$ of matrix $\mathbf{M}$ are, the more similar the n ratios Ψ_j will be.

Equation (9) shows that the $(1+\Psi_j)$ can be made dependent on an *economy-wide average* $(1 + \psi)$ and, as in (A.30), a 'covariance-plus-simple average' decomposition of an industry-level weighted average:

$$(1 + \Psi_j) = (1 + \psi) \sum_{i=1}^{n} \sigma_i m_{ij} = (1 + \psi) \left[n\text{Cov}(\sigma_i, m_{ij}) + \bar{\sigma} \right] \text{ for } j \in N, \tag{A.31}$$

where $\bar{\sigma} \equiv \frac{1}{n} \sum_{i=1}^{n} \sigma_i = \frac{1+\bar{\psi}}{1+\psi}$. The distance of $\bar{\sigma}$ from 1 measures the difference in industries weights from uniform weights $\frac{1}{n}$.

B Construction of the database

B.1 The World Input-Output Database

The World Input-Output Database (WIOD), in its 2016 release, constructed by Timmer et al. (2015), provides estimates of annual time series of input–output tables (IOTs) covering 43 countries — 28 EU countries and 15 other major countries in the world — for the period from 2000 to 2014. The sample of countries represents up to 86% of the world economy in 2016 and includes rich- and middle-income economies with different industrial structures and development profiles — least developed countries are not included in the sample. The list of the 43 economies, and their acronyms (in parenthesis) used throughout the document, is given in Table B.1. We have decided to exclude Malta from the sample.

Australia (AUS) [49]	Austria (AUT) [54]	Belgium (BEL) [54]	Bulgaria (BGR) [54]
Brazil (BRA) [47]	Canada (CAN) [51]	Switzerland (CHE) [49]	China, People's Republic of (CHN) [47]
Cyprus (CYP) [54]	Czech Republic (CZE) [54]	Germany (DEU) [54]	Denmark (DNK) [54]
Spain (ESP) [54]	Estonia (EST) [54]	Finland (FIN) [54]	France (FRA) [54]
United Kingdom of Great Britain and Northern Ireland (GBR) [54]	Greece (GRC) [54]	Croatia (HRV) [54]	Hungary (HUN) [54]
Indonesia (IDN) [47]	India (IND) [45]	Ireland (IRL) [54]	Italy (ITA) [54]
Japan (JPN) [50]	Republic of Korea (KOR) [53]	Lithuania (LTU) [54]	Luxembourg (LUX) [52]
Latvia (LVA) [54]	Mexico (MEX) [52]	Malta (MLT) [52]	Netherlands (NLD) [54]
Norway (NOR) [54]	Poland (POL) [54]	Portugal (PRT) [54]	Romania (ROU) [54]
Russian Federation (RUS) [33]	Slovakia (SVK) [54]	Slovenia (SVN) [54]	Sweden (SWE) [53]
Turkey (TUR) [46]	Taiwan (TWN) [54]	United States (USA) [54]	

Table B.1: List of the 43 countries included in the WIOD database, 2016 release, and number of industries for which there is information available in each country-year.

The basis for the construction of the IOTs are the Supply and Use Tables (SUTs) obtained from official national sources and adapted to a common disaggregation level of 56 industries based on the System of National Accounts 2008 (SNA2008) framework. The input years and the number of releases for which SUTs are available are uneven and scattered with the base methodology based on the SNA2008, SNA1993, and International System of Industrial Classification, Revision 3, frameworks. This information is used to construct the world industry-by-industry IOTs from which we obtain the IOTs for each country-year.

The WIOD includes fictitious industries that are statistical artifacts to balance the tables. We decide to omit industries T ("Activities of households as employers; undifferentiated goods- and services- producing activities of households for own use") and U ("Activities of extraterritorial organizations and bodies"), whose entries are mostly zeros. Table B.2 shows the list of the final 54 industries. However, not every country in the WIOD has information for the 54 industries. *Our calculations do not take into account the zeros from these industries.* The number of industries with non-zero information for each country is given in square brackets in Table B.1. The number of countries with information for each industry is given in square brackets in Table B.2. The country with the least information is Russia (with 31 industries), but most of the economies fluctuate between 52 and 54 industries.

ISIC4 code	Sector description
A01	Crop and animal production, hunting and related service activities
A02	Forestry and logging
A03	Fishing and aquaculture
B	Mining and quarrying
C10-C12	Manufacture of food products, beverages and tobacco products
C13-C15	Manufacture of textiles, wearing apparel and leather products
C16	Manufacture of wood and of products of wood and cork, except furniture; straw and plaiting materials
C17	Manufacture of paper and paper products
C18	Printing and reproduction of recorded media
C19	Manufacture of coke and refined petroleum products
C20	Manufacture of chemicals and chemical products
C21	Manufacture of basic pharmaceutical products and pharmaceutical preparations
C22	Manufacture of rubber and plastic products
C23	Manufacture of other non-metallic mineral products
C24	Manufacture of basic metals
C25	Manufacture of fabricated metal products, except machinery and equipment
C26	Manufacture of computer, electronic and optical products
C27	Manufacture of electrical equipment
C28	Manufacture of machinery and equipment n.e.c.
C29	Manufacture of motor vehicles, trailers and semi-trailers
C30	Manufacture of other transport equipment
C31-C32	Manufacture of furniture; other manufacturing
C33	Repair and installation of machinery and equipment
D35	Electricity, gas, steam and air conditioning supply
E36	Water collection, treatment and supply
E37-E39	Sewerage; waste collection, treatment and disposal activities; materials recovery; other waste services
F	Construction
G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
G46	Wholesale trade, except of motor vehicles and motorcycles
G47	Retail trade, except of motor vehicles and motorcycles
H49	Land transport and transport via pipelines
H50	Water transport
H51	Air transport
H52	Warehousing and support activities for transportation
H53	Postal and courier activities
I	Accommodation and food service activities
J58	Publishing activities
J59-J60	Motion picture, video and television production, sound and music; programming and broadcasting
J61	Telecommunications
J62-J63	Computer programming, consultancy and related activities; information service activities
K64	Financial service activities, except insurance and pension funding
K65	Insurance, reinsurance and pension funding, except compulsory social security
K66	Activities auxiliary to financial services and insurance activities
L68	Real estate activities
M69-M70	Legal and accounting activities; activities of head offices; management consultancy activities
M71	Architectural and engineering activities; technical testing and analysis
M72	Scientific research and development
M73	Advertising and market research
M74-M75	Other professional, scientific and technical activities; veterinary activities
N	Administrative and support service activities
O84	Public administration and defence; compulsory social security
P85	Education
Q	Human health and social work activities
R-S	Other service activities

Table B.2: List with the 54 industries from the WIOD, 2016 release, considered in the sample. *Notes:* Omitted industries are 'Activities of households as employers; undifferentiated goods- and services- producing activities of households for own use' (T) and 'Activities of extraterritorial organisations and bodies' (U).

The WIOD also provides the Socio-Economic Accounts (SEAs) which provide industry-level data on the use of primary inputs (capital and labor), intermediate inputs, gross output, and the components of value added at current and constant

Symbol	Variable	Units
GO	Gross output by industry at current basic prices	in millions of national currency
II	Intermediate inputs at current purchasers' prices	in millions of national currency
VA	Gross value added at current basic prices	in millions of national currency
EMP	Number of persons engaged	thousands
EMPE	Number of employees	thousands
H_EMPE	Total hours worked by employees	millions
COMP	Compensation of employees	in millions of national currency
LAB	Labour compensation	in millions of national currency
CAP	Capital compensation	in millions of national currency
$\mathcal{K}$	Nominal capital stock	in millions of national currency
GO_PI	Price levels gross output	2010=100
II_PI	Price levels of intermediate inputs	2010=100
VA_PI	Price levels of gross value added	2010=100
GO_QI	Gross output, volume indices	2010=100
II_QI	Intermediate inputs, volume indices	2010=100
VA_QI	Gross value added, volume indices	2010=100

Table B.3: List of the variables in the Socio-Economic Accounts in the WIOD database, 2016 release, and their description.

prices, using the same classification system as the WIOTs. Table B.3 provides a description of the information available. A comprehensive overview of the sources and methodological choices for the original release can be found in Dietzenbacher et al. (2013). The labor compensation includes wages and salaries paid to employees and the self-employed in each industry, whose earnings are imputed using the average wages prevailing in the sector (Dietzenbacher et al., 2013).

The industry-by-industry tables are in current market international dollar prices. The value data of the SAEs are denoted in millions of national currency. Values were converted to dollars using the exchange rates provided in the WIOD as a separate file.

B.2 The methodology to compute the total profits-wages ratios and total capital intensities

While the *direct* measures of labor costs, profits, capital, and labor can be calculated directly from the data from the WIOD, we employed the same approach as in (A.24) to obtain their total or vertically integrated counterparts —that is, $\boldsymbol{\omega}'\boldsymbol{\Lambda} \equiv \boldsymbol{\Omega}' \equiv [\Omega_j]$ and $\boldsymbol{\pi}'\boldsymbol{\Lambda} \equiv \boldsymbol{\Pi}' \equiv [\Pi_j]$.

Leontief input-coefficients matrix and inverse. For each country-year we construct matrix $\mathbf{Z} \equiv [z_{ij}]$ by summing industry j's purchases of domestic z_{ij}^D and imported z_{ij}^M intermediate inputs from industry i at market prices, $z_{ij} = z_{ij}^D + z_{ij}^M$. Then, Leontief's input-coefficients matrix $\mathbf{A} \equiv [a_{ij}] \geqq \mathbf{0}$ is constructed assuming proportionality of inputs z_{ij} and outputs x_j, $a_{ij} = \frac{z_{ij}}{x_j}$, for $i, j \in N$. Finally, Leontief's inverse $\mathbf{\Lambda} \equiv [\Lambda_{ij}]$ is obtain as $\mathbf{\Lambda} \equiv (\mathbf{I} - \mathbf{A})^{-1}$.[B.1]

Unit labor costs (ω_j, Ω_j) **and unit profits** (π_j, Π_j)**.** Direct unit labor costs and unit profits for industry j for each country-year are obtained by dividing the total labor compensation (LAB) and total capital compensation (CAP) by gross output (GO) for each industry: $\omega_j = \frac{\text{LAB}_j}{\text{GO}_j}$ and $\pi_j = \frac{\text{CAP}_j}{\text{GO}_j}$. For each country-year we construct vectors $\boldsymbol{\omega}' \equiv [\omega_j]$ and $\boldsymbol{\pi}' \equiv [\pi_j]$. The vectors with the *total* unit labor costs and unit profits are obtain as $\boldsymbol{\omega}'\mathbf{\Lambda} \equiv \mathbf{\Omega}' \equiv [\Omega_j]$ and $\boldsymbol{\pi}'\mathbf{\Lambda} \equiv \mathbf{\Pi}' \equiv [\Pi_j]$, respectively.

Profit-wages ratios (ψ_j, Ψ_j)**.** Let $\hat{\mathbf{y}} = \text{diag}\{y_1, \ldots, y_n\}$ for any vector $\mathbf{y}' \equiv [y_j]$. Then, for each country-year, the vectors with industries' direct and total profits-wages ratios are obtained as $\boldsymbol{\pi}'\hat{\boldsymbol{\omega}}^{-1} \equiv \boldsymbol{\psi}' \equiv [\psi_j] \equiv [\frac{\pi_j}{\omega_j}]$ and $\mathbf{\Pi}'\hat{\mathbf{\Omega}}^{-1} \equiv \mathbf{\Psi}' \equiv [\Psi_j] \equiv [\frac{\Pi_j}{\Omega_j}]$, respectively.

Capital intensities. For each country-year, the direct unit capital in industry j is obtained by aggregating the value of intermediate inputs (II) and the nominal stock of capital ($\mathcal{K}$) and then dividing it by gross output (GO), $\kappa_j = \frac{\text{II}_j + \mathcal{K}_j}{\text{GO}_j}$. The direct unit labor in industry j for each country-year is obtained dividing the number of persons engaged (EMP) by gross output (GO), $l_j = \frac{\text{EMP}_j}{\text{GO}_j}$. For each country-year we construct vectors $\boldsymbol{\kappa}' \equiv [\kappa_j]$ and $\mathbf{l}' \equiv [l_j]$. The vectors with the total unit capital and total unit labor are obtain as $\boldsymbol{\kappa}'\mathbf{\Lambda} \equiv \mathbf{K}' \equiv [K_j]$ and $\mathbf{l}'\mathbf{\Lambda} \equiv \mathbf{v}' \equiv [v_j]$, respectively. Finally, industries' total capital intensities are obtained as $\frac{K_j}{v_j}$ for $j \in N$.

[B.1] In each case $(\mathbf{I} - \mathbf{A})^{-1} \geqslant \mathbf{0}$.

C Empirical distribution of capital intensities

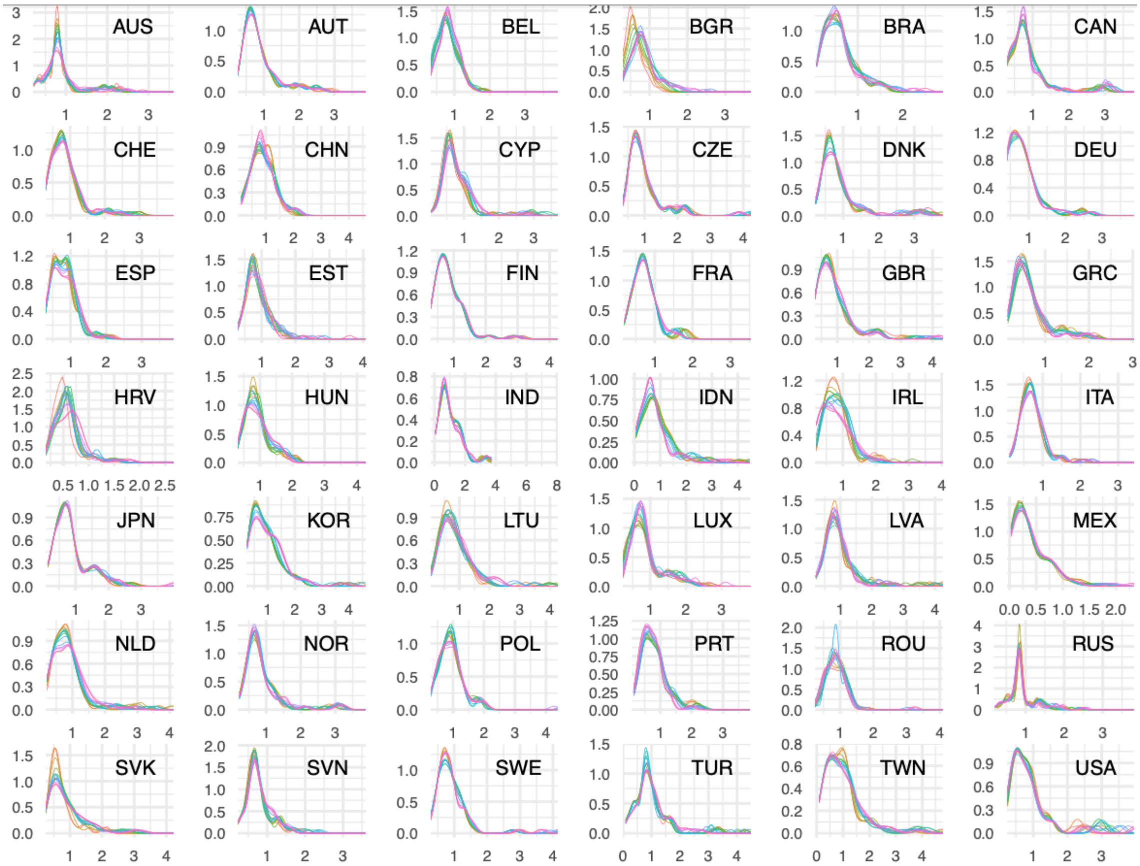

Figure C.1: Total capital intensities of industries within countries. *Source*: authors' calculations using the World Input-Output Database.

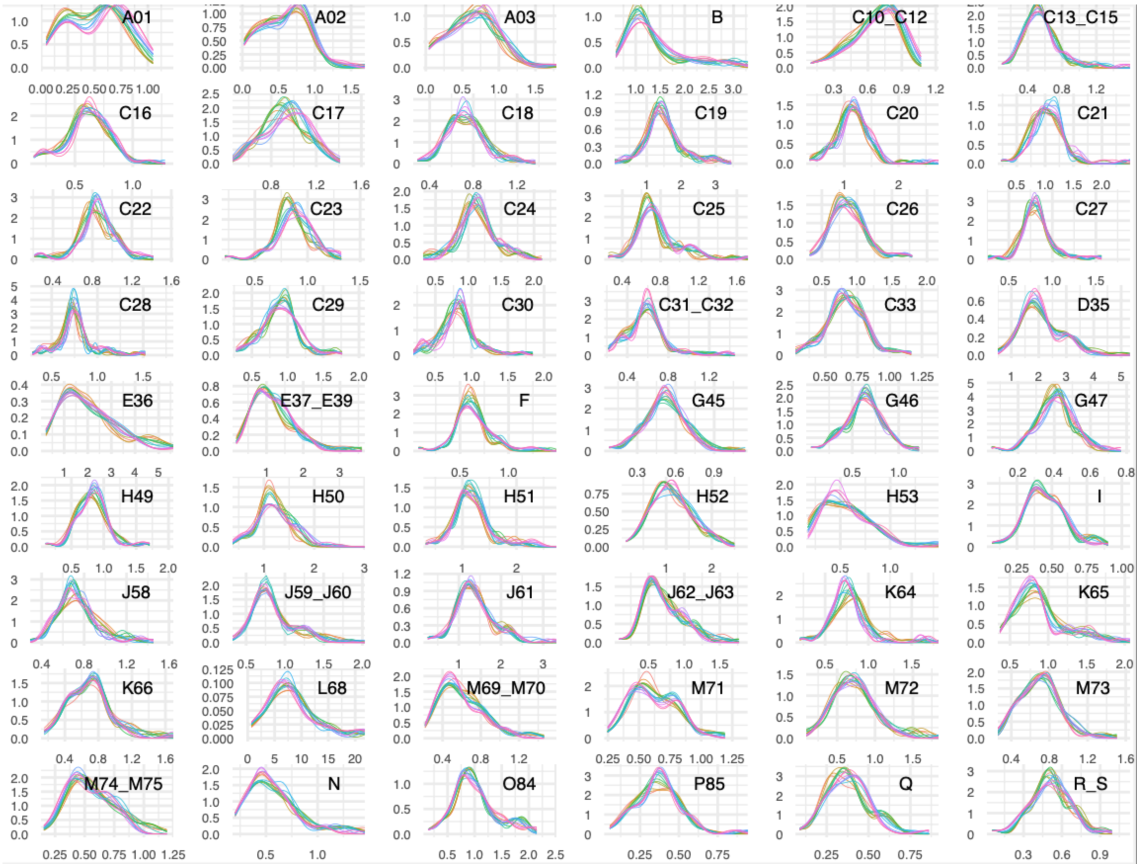

Figure C.2: Total capital intensities of industries across countries. *Source*: authors' calculations using the World Input-Output Database.

D Details on the Bayesian methodology for testing the Hypothesis of Ψ_j^A and Ψ_k^B mean-overlap

D.1 The Bayesian approach

The Bayesian approach to statistical inference defines probabilities as the theoretical likelihoods or "degrees of belief" regarding the frequencies with which outcomes are observed. This implies treating parameters as random variables and the data as fixed, which is the opposite approach to standard frequentist inference.

Drawing on basic probability theory, Bayes' rule provides a straightforward way to find the *posterior* probability distribution $p(\theta|y)$ of a parameter θ given the known value of the data y by multiplying the *prior* probability of the parameter θ and the conditional probability distribution $p(y|\theta)$, known as the likelihood function. The latter encapsulates the key distributional assumption regarding the data generating process, and the former is our prior knowledge about the parameters. Dividing by the marginal distribution $p(y)$ we obtain the posterior density properly, but the unnormalized posterior density $p(\theta|y) \propto p(\theta)p(y|\theta)$ is sufficient.

This framework offers considerable modeling flexibility, an intuitive interpretation of estimation uncertainty, and, most importantly, a parsimonious way of exploiting prior information (McElreath, 2020). The posterior probability distribution is a weighted average of the likelihood and the prior, with the importance of the prior decreasing substantially along with the uncertainty as more data become available. Therefore, the region populated by draws from the posterior distribution of any parameter represents the credible prediction interval where the mean of the actual distribution could be given the information contained in the original data.

D.2 Effect sizes of the different means

Following Kruschke (2013), we compute the effect size as $(\mu_1 - \mu_2)/\sqrt{(\sigma_1^2 + \sigma_2^2)/2}$. This is a simplification that, in the author's opinion, makes no relevant difference to the standard formula that accounts for the difference in sample size of the groups, which in this case are almost equal in all cases.

D.3 The Gamma model for Ψ_j

The Gamma model consist on

$$\Psi_j|\alpha, \beta \sim \mathbf{Gamma}(\alpha, \beta) \tag{D.1}$$

$$\alpha \sim \mathbf{Normal}(0, 10) \tag{D.2}$$

$$\beta \sim \mathbf{Normal}(0, 10) \tag{D.3}$$

Following the literature, we assign weakly informative priors to the parameters (α, β) which do not affect the results but help to model convergence (Gelman et al., 2013).

The full implementation of the Gamma model is done by posterior simulation using the Hamiltonian Monte Carlo (HMC) algorithm via *Stan* (Team, 2022).[D.1] The model runs 4 Markov chains with 3000 iterations each, of which 1000 are warm-up iterations. The model converges and shows no pathological behavior that could compromise the results of the estimation —for instance, more than 5% divergent transitions or bad Pareto k-factors. The scale reduction factor ($\hat{R}$) and the bulk Effective Sample Size (ESS) are below 1.01 and safely above the recommended 400 samples, respectively.

D.4 Table 1 for all countries, 2014: 95% highest density credibility bound of $\mu^{\alpha} - \mu^{\beta}$

[D.1] For a more general introduction see Betancourt (2018)

	AUS	AUT	BEL	BGR	BRA	CAN	CHE	CHN	CYP	CZE	DEU	DNK	ESP	EST	FIN	FRA	GBR	GRC	HRV	HUN
AUT	0.03																			
BEL	0.2																			
BGR			0.13																	
BRA	0.36	0.55	0.68	0.37																
CAN			0.14		0.38															
CHE	0.41	0.15	0.01	0.31	0.83	0.34														
CHN		0.11	0.29		0.31		0.51													
CYP					0.46		0.13													
CZE		0.17	0.36		0.29		0.6		0.05											
DEU	0.1			0.03	0.6	0.05	0.08	0.18		0.24										
DNK			0.01		0.39		0.18													
ESP			0.11		0.36		0.29				0.02									
EST		0.14	0.32		0.27		0.54		0.03		0.21									
FIN	0.11			0.05	0.6	0.05	0.01	0.18		0.23			0.03	0.21						
FRA	0.28	0.05		0.2	0.73	0.22		0.36	0.04	0.42		0.1	0.19	0.39						
GBR	0.26	0.02		0.18	0.72	0.2		0.34	0.02	0.4		0.07	0.17	0.37						
GRC	0.33	0.51	0.64	0.35		0.36	0.78	0.28	0.44	0.27	0.57	0.37	0.34	0.26	0.57	0.69	0.68			
HRV	0.13			0.07	0.62	0.08		0.2		0.25			0.06	0.23				0.59		
HUN		0.1	0.28		0.33		0.51				0.17				0.17	0.36	0.34	0.31	0.19	
IDN	0.21	0.44	0.61	0.22		0.24	0.81	0.15	0.32	0.14	0.51	0.24	0.21	0.11	0.5	0.67	0.65		0.51	0.18
IND	0.56	0.8	0.99	0.55		0.57	1.24	0.5	0.65	0.51	0.88	0.54	0.53	0.45	0.85	1.03	1.02		0.84	0.54
IRL		0.04	0.19		0.23		0.36				0.1				0.11	0.26	0.23	0.22	0.14	
ITA			0.12		0.32		0.29				0.03				0.05	0.2	0.17	0.3	0.08	
JPN			0.13		0.31		0.31				0.05				0.06	0.21	0.19	0.29	0.08	
KOR					0.51		0.14	0.06		0.11				0.09		0.05	0.02	0.48		0.05
LTU	0.2	0.43	0.6	0.22		0.23	0.8	0.15	0.31	0.13	0.5	0.23	0.21	0.11	0.49	0.66	0.64		0.5	0.18
LUX	0.27	0.04		0.2	0.72	0.21		0.34	0.04	0.41		0.09	0.18	0.38				0.69		0.34
LVA		0.14	0.33		0.3		0.56		0.03		0.22				0.21	0.39	0.37	0.28	0.23	
MEX	1.81	1.97	2.09	1.8	0.99	1.82	2.23	1.78	1.86	1.78	2.01	1.78	1.78	1.75	2	2.12	2.11	0.9	1.99	1.81
MLT	0.07				0.58	0.01	0.14	0.15		0.21				0.18		0.03	0.01	0.54		0.14
NLD					0.51		0.17	0.06		0.11				0.09		0.07	0.05	0.48		0.05
NOR			0.01		0.45		0.19			0.03				0.01		0.1	0.07	0.43		
POL	0.16	0.41	0.6	0.18	0.06	0.19	0.83	0.1	0.28	0.09	0.49	0.19	0.16	0.06	0.47	0.65	0.63	0.04	0.48	0.14
PRT	0.03	0.25	0.39	0.05		0.07	0.55		0.16		0.31	0.09	0.06		0.31	0.45	0.43		0.33	
ROU	0.54	0.75	0.9	0.54		0.56	1.08	0.49	0.63	0.48	0.81	0.55	0.53	0.46	0.8	0.95	0.93		0.8	0.52
RUS	0.27	0.03		0.19	0.74	0.21		0.36	0.02	0.44		0.08	0.18	0.4				0.7		0.36
SVK	0.24	0.47	0.64	0.25		0.27	0.83	0.18	0.35	0.17	0.54	0.27	0.25	0.14	0.53	0.69	0.67		0.54	0.22
SVN	0.23			0.16	0.69	0.17		0.31		0.37		0.05	0.14	0.34				0.66		0.3
SWE		0.05	0.22		0.36		0.44				0.12				0.13	0.3	0.27	0.34	0.14	
TUR	1.17	1.35	1.49	1.16	0.35	1.19	1.64	1.13	1.24	1.13	1.41	1.16	1.15	1.1	1.39	1.52	1.51	0.28	1.38	1.16
TWN					0.5			0.04		0.08				0.07				0.5		0.04
USA		0.11	0.28		0.26		0.47		0.01		0.18				0.18	0.35	0.32	0.24	0.21	

Table D.1: 95% credibility bound of the pair-wise difference of posterior distributions of the mean parameters of the total profit-wage ratios for all countries ($\mu^{\alpha} - \mu^{\beta}$). Missing values indicate a possible overlap with a lower than 95% credibility interval for the null. *Source*: authors' calculations.

	IDN	IND	IRL	ITA	JPN	KOR	LTU	LUX	LVA	MEX	MLT	NLD	NOR	POL	PRT	ROU	RUS	SVK	SVN	SWE	TUR
IND	0.04																				
IRL	0.06	0.35																			
ITA	0.17	0.47																			
JPN	0.15	0.46																			
KOR	0.39	0.73																			
LTU		0.04	0.05	0.16	0.15	0.38															
LUX	0.65	1.02	0.25	0.19	0.21	0.04	0.65														
LVA	0.14	0.51				0.09	0.14	0.38													
MEX	1.43	1.25	1.64	1.73	1.72	1.92	1.42	2.11	1.78												
MLT	0.49	0.86	0.07		0.02		0.48	0.02	0.19	2.01											
NLD	0.39	0.74					0.39	0.06	0.09	1.93											
NOR	0.31	0.64					0.31	0.09	0.01	1.86											
POL		0.19		0.11	0.09	0.35		0.64	0.09	1.56	0.46	0.35	0.27								
PRT		0.05		0.01	0.01	0.21		0.44		1.39	0.28	0.2	0.14								
ROU	0.1		0.39	0.48	0.47	0.7	0.09	0.94	0.49	1.03	0.79	0.7	0.62	0.21	0.12						
RUS	0.68	1.08	0.25	0.18	0.2	0.02	0.68		0.41	2.14	0.01	0.05	0.08	0.68	0.45	0.97					
SVK			0.1	0.2	0.18	0.42		0.68	0.18	1.38	0.51	0.42	0.35			0.05	0.71				
SVN	0.62	0.99	0.21	0.16	0.16		0.61		0.34	2.09		0.03	0.05	0.61	0.41	0.91		0.65			
SWE	0.21	0.57					0.21	0.28		1.82	0.09			0.17	0.04	0.55	0.3	0.25	0.25		
TUR	0.76	0.56	1.01	1.1	1.09	1.3	0.77	1.51	1.14	0.24	1.39	1.31	1.24	0.89	0.76	0.36	1.54	0.72	1.48	1.19	
TWN	0.32	0.57	0.03				0.33		0.06	1.8				0.27	0.22	0.61		0.36			1.2
USA	0.09	0.42				0.07	0.09	0.33		1.71	0.15	0.06		0.03		0.43	0.35	0.12	0.3		1.07

Table D.2: 95% credibility bound of the pair-wise difference of posterior distributions of the mean parameters of the total profit-wage ratios for all countries ($\mu^\alpha - \mu^\beta$). Missing values indicate a possible overlap with a lower than 95% credibility interval for the null. *Source*: authors' calculations.

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