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On the Over-determination Problem in a Two Sector Neo-Kaleckian Model

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Abstract

In this paper, we aim to solve the over-determination problem in two-sector neo-Kaleckian models raised by Park (1995) against Dutt (1990). After summarising the over-determination problem and existing solutions, we argue that the over-determination problem is not caused by the incompatibility of sectors' investment functions and equalised rates of profit, but rather the incompatibility of profit rate equalisation and the arbitrarily given mark-up rates of different sectors. We propose to solve the problem by introducing an endogenous variable, the relative mark-up ratio, which makes the model perfectly determined and more logically consistent. We also discuss the adjustment mechanism from the short-run to the long-run equilibrium.

Keywords: two-sector neo-Kaleckian model; equalised profit rate; mark- up pricing; free competition

JEL Codes: B51; O41; E11

1. Introduction*

The neo-Kaleckian theory of growth and distribution, put forward by Rowthorn (1981), Taylor (1983), Dutt (1984), and Amedeo (1986), and later developed by Kurz (1990), Bhaduri and Marglin (1990), has become a powerful alternative to the neo-classical growth theory. Due to its flexibility and ability to incorporate other heterodox economic perspectives, neo-Kaleckian theory has been widely generalised and used in empirical research. Among these generalisations, Dutt (1987, 1988, 1990) and Lavoie and Ramirez-Gaston (1997) extend a one-sector model into a two-sector model. Recent development and generalisations of the two-sector model include analyses of long-run convergence

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(Kim and Lavoie, 2017), the introduction of intermediate goods (Fujita, 2019), and endogenous labour productivity growth (Nishi, 2020).

Despite its popularity, the neo-Kaleckian model is subject to several criticism. A persistent critique is the divergence of the actual rate of capacity utilisation from the normal rate, even in the long-run period, and the related stability problem. Hein *et al*. (2011, 2012) survey critiques and solutions around these debates (see also Nikiforos, 2016). As concerns the two-sector model, Park (1995) raises the critique that Dutt's model suffers from an over-determination problem after integrating sectoral investment functions. Dutt (1997) refutes Park and insists that Park's incorrect investment functions for sectors actually led to the over-determination. Following this Dutt-Park controversy, Araujo and Drumond (2021) propose to solve the over-determination problem by introducing investment allocation.

In this paper, we seek to solve the over-determination problem from another perspective; that is, we introduce sector-relative mark-up ratio. We can summarise our argument as follows: what really matters for the over- determination problem is not the incompatibility of the sectoral investment functions and steady state characterised by the equalised rate of profit and growth rate. What is incompatible with the equalised rate of profit is the arbitrarily and independently given mark-up rates of different sectors. After endogenising the relationship between the mark-up rates in different sectors, the over-determination problem does not arise, and the model is more logically consistent. To maintain focus on the over-determination problem, we do not discuss other issues like the relationship between the normal and actual utilisation rates. This of course does not mean that these issues are not important.

This paper proceeds as follows. In Section 2, we summarise the model in dispute in the Dutt-Park controversy to grasp the core of the over-determination problem. Section 3 makes some comments on the existing solutions to the over-determination problem. Section 4 describes the relationships amongst mark-up pricing, classical free competition, and the equalised profit rate. In Section 5, we introduce the sector relative mark-up ratio into the model and show how it solves the over-determination problem. Section 6 discusses how the short-run equilibrium adjusts to the long-run equilibrium determined in Section 5. Section 7 provides some concluding remarks.

2. The over-determination problem

To grasp the main idea of the over-determination problem, we first restate the two-sector model in Dutt (1987, 1990, 1995) and Park (1995, 1998). The economy has two sectors, with sector 1 producing consumption goods and sector 2 producing investment goods. Both sectors use only labour and capital and produce goods with a linear technology[1]. Capital does not depreciate. The economy has two classes: capitalists and workers. The former make investments, with the assumption that they do not consume for simplicity; that is, their saving rate is 1[2]. The latter spend all their wages on consumption goods, with the assumption of a uniform wage rate. We can represent the two-sector model using the following equations:

$$p_1 = p_2 \left(\frac{v_1}{u_1}\right) r_1 + W l_1 \tag{1a}$$

$$p_2 = p_2 \left(\frac{v_2}{u_2}\right) r_2 + W l_2 \tag{1b}$$

$$1 = w(l_1 + l_2 x) \tag{1c}$$

$$x = g_1 \frac{v_1}{u_1} + g_2 \frac{v_2 x}{u_2} \tag{1d}$$

$$W = w p_1 \tag{1e}$$

$$p_1 = (1 + \tau_1) W l_1 \tag{1f}$$

$$p_2 = (1 + \tau_2) W l_2 \tag{1g}$$

$$g_1 = g_1(u_1, r_1) \tag{1h}$$

$$g_2 = g_2(u_2, r_2) \tag{1i}$$

$$r_1 = r_2 = r \tag{1j}$$

$$g_1 = g_2 = g \tag{1k}$$

In System (1), p_i is the price of commodity i, and v_i and l_i are the constant capital-output ratio and labour-output ratio, respectively. u_i is the rate of capacity utilisation, and W and w are the money wage rate and real wage rate in terms of consumption goods, respectively. r_i is the rate of profit, g_i is the rate of capital accumulation, and τ_i is the mark-up rate. Let X_i and K_i be the output and capital stock in sector i; then, $X_i = u_i \frac{K_i}{v_i}$. $x = \frac{X_2}{X_1}$ is the ratio of the investment good's output relative to the consumer good's output.

Equations (1a) and (1b) state that the revenues (for producing 1 unit commodity) in both sectors are divided between wages and profits (where $p_2(\frac{v_i}{u_i})r_i = p_2(\frac{r_i K_i}{X_i})r_i$). Equations (1c) and (1d) are the commodity balance equations indicating that workers consume the commodities produced in sector 1, while the commodities produced by sector 2 are used as investment goods. Equation (1e) is the real wage equation. Equations (1f) and (1g) are the sectors' mark-up pricing equations. Equations (1h) and (1i) are the sectors' investment functions. Equations (1j) and (1k) define the long-run steady state of the economy, where Equation (1j) is the classical competition characterised by profit rate equalisation, and

[1] A feature of modern economies is that commodities are produced by means of commodities; that is, production processes use produced inputs. However, to focus on the over-determination problem, we do not introduce intermediate goods into the model. For the problems that arise in neo-Kaleckian theory when considering produced inputs, see Steedman (1992, 1999), and the following debate around Kaleckian models (Sawyer, 1992; Mainwaring, 1992; Kriesler, 1992, 1993; Steindl, 1993; Steedman, 1993, 1999).

[2] One can assume that a saving rate of less than 1 for capitalists, but this does not change our results.

Equation (1k) is the balanced growth equation. System (1) has 10 variables p_i, w, r_i, u_i, g_i, and x and 11 equations; therefore, the model is over-determined, as Park (1995) argues. Although System (1) has 11 equations, they are not determined simultaneously. We can eliminate some equations and variables to clarify the over-determination problem. First, p_1, p_2, w, and x can be determined by (1e), (1f), (1g), and (1c), and we can eliminate these 4 equations and take p_1, p_2, w, and x as given for the remainder. Second, we can present u_i as a linear function of r_i based on Equation (1a), (1b):

$$u_1 = \frac{pv_1}{\pi_1} r_1 = \frac{1+\tau_1}{\tau_1} pv_1 r_1 = \frac{1+\tau_2}{\tau_1}\frac{l_2}{l_1} v_1 r_1 \tag{2a}$$

$$u_2 = \frac{v_2}{\pi_2} r_2 = \frac{1+\tau_2}{\tau_2} v_2 r_2 \tag{2b}$$

where $p = \frac{p_2}{p_1}$ and π_i is the profit share in sector i.

We can substitute u_i into the other equations and eliminate Equations (1a) and (1b). Then, the remaining variables are g_i and r_i, and the equations which matter are (1d), (1h), (1i), (1j), and (1k). Further, considering $X_i = u_i \frac{K_i}{v_i}$, we can transform Equation (1d) into:

$$g_1 K_1 + g_2 K_2 = r_1 K_1 + r_2 K_2 \tag{3}$$

If we assume that the investment functions (1h) and (1i) are linear[3]:

$$g_1 = \alpha_1 + \beta_1 r_1 + \gamma_1 u_1 \tag{4}$$

$$g_2 = \alpha_2 + \beta_2 r_2 + \gamma_2 u_2 \tag{5}$$

Then, together with System (2), Equations (1h) and (1i) become:

$$g_1 = \alpha_1 + (\beta_1 + \gamma_1 \frac{1+\tau_2}{\tau_1}\frac{l_2}{l_1} v_1) r_1 \tag{6}$$

$$g_2 = \alpha_2 + (\beta_2 + \gamma_2 \frac{1+\tau_2}{\tau_2} v_2) r_2 \tag{7}$$

We observe that for System (1), what really matters for the over-determination problem are Equations (1j), (1k), (3), (6), and (7). In the following, we refer to these equations as the core equations to the over-determination problem. We now have 5 equations and 4 variables (g_i and r_i). Further, if (1j) and (1k) hold, then we can simplify these equations as follows:

$$g = r \tag{8a}$$

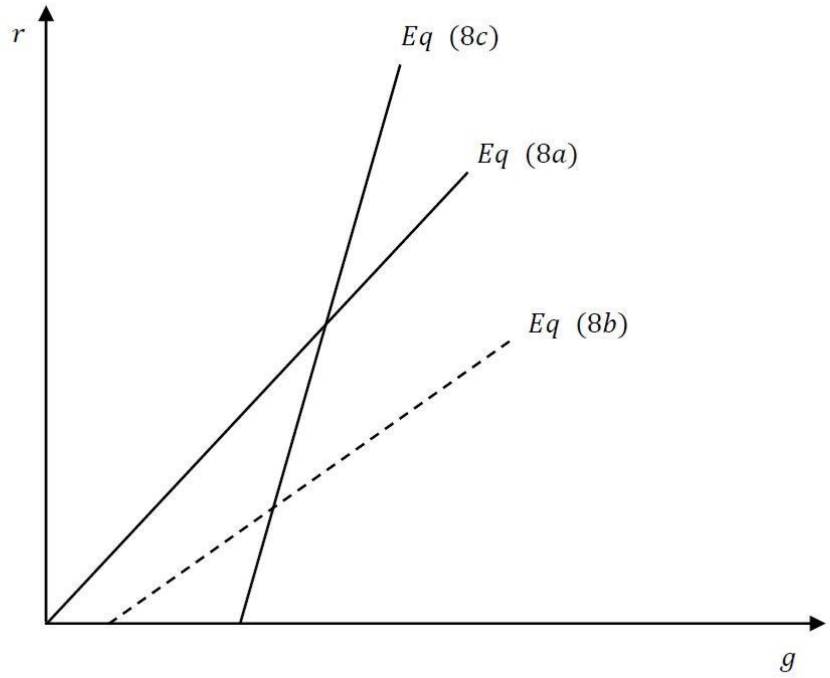
$$g = \alpha_1 + (\beta_1 + \gamma_1 \frac{1+\tau_2}{\tau_1}\frac{l_2}{l_1} v_1) r \tag{8b}$$

$$g = \alpha_2 + (\beta_2 + \gamma_2 \frac{1+\tau_2}{\tau_2} v_2) r \tag{8c}$$

[3] To focus on the over-determination problem, we use a Dutt-Rowthorn type investment function. However, the logic of the over-determination problem does not change if we adopt a Bhaduri-Marglin type investment function (Bhaduri and Marglin, 1990).

System (8) represents three straight lines in the (g, r) quadrant (illustrated in Figure 1), where the case in which these lines have one intersection can hold only by fluke. Therefore, we find over-determination in general.

Figure 1. Illustration of the over-determination problem



3. Existing solutions

Mathematically, there are two ways to solve the over-determination problem. The first is to reduce an equation without changing the number of unknown variables, and the second is to add another endogenous variable without changing the number of equations. In this section, we discuss the existing solutions to the over-determination problem based on these two categories.

In the first category, we can eliminate one of the core equations (Equations (1j), (1k), (3), (6), and (7)). Dutt (1987, 1990) and Kim and Lavoie (2017, adjustment mechanism 2) fit roughly into this category. Dutt (1990) argues that Park's sectoral investment functions generally contradict classical competition, and instead of using sectoral investment functions, Dutt (1987, 1990) uses an aggregate investment function with the following form:

$$g = g(r, u_1, u_2) \tag{9}$$

If we replace Equations (1h) and (1i) with Equation (9), then for System (1), we have 10 variables and 10 equations, and the over-determination problem does not arise. Taking System (2) into consideration, Equation (9) is equivalent to assuming that the rate of accumulation is a function of the profit rate, $g(r)$. Hence, we reduce Equations (8b) and (8c) to one equation, and the system is perfectly determined.

Dutt (1997) also argues that the classical competition and sectoral investment functions can be compatible, but Park's sectoral investment functions should be replaced: the differences between the rates of accumulation of different sectors should be governed by profit-rate differentials; that is, $g_1 - g_2 = \lambda(r_1 - r_2)$, where λ is a positive coefficient. Kim and Lavoie (2017) also adopt this sectoral investment function in one of their proposed long- run dynamic adjustment mechanisms (see Kim and Lavoie (2017), Equation (18.2) on page 190). Since in the long-run steady state $r_1 = r_2$ holds, $g_1 = g_2$ also holds. Therefore, the sectoral investment functions in Dutt (1997) are equivalent to the assumption of an aggregate investment function.

Similarly, if we retain the sectoral investment functions and eliminate another equation from the system, for instance, if Equation (1j) (or (1k)) does not hold, meaning that the rates of profit (or accumulation) are not equalised in the steady state, then the system is also perfectly determined.

The above solutions appear to suggest that in the two-sector neo-Kaleckian model, either the equalisation of the profit rate does not hold in the steady state, or if the profit rates are equalised, the sectoral investment functions (Equations (1h) and (1i)) are not compatible with it. As Dutt (1997) argues, the Kalecki-Steindl authors would either jettison the validity of the classical rate of profit equalisation or introduce classical competition directly into the investment function. However, Park (1995) criticises these solutions. Specifically, for the aggregate investment function, it is odd to refer to investment behaviour alone in aggregate terms considering that mark-up rates, prices, and other factors are given in sectoral terms. Additionally, the non-uniform rates of profit are incompatible with the steady state because if profit rates are not uniform, then productive resources must be removed between sectors (a similar argument applies to the case when Equation (1k) does not hold).

In the second category, (at least) one endogenous variable is added to the core equations. Kim and Lavoie (2017) and Araujo and Drumond (2020) can be roughly grouped into this category. Kim and Lavoie (2017) use target-return pricing instead of mark-up pricing, but these two pricing theories are the same under constant target rates of return. In Regime 3 proposed by Kim and Lavoie (2017), the target rates of return adjust to the actual rates of profit. Due to this mechanism, in the long-run steady state, both the profit rates and growth rates in different sectors are equalised. Compared with the model in the Dutt-Park controversy, the relative prices in Kim and Lavoie (2017) are not given, but endogenous. Araujo and Drumond (2020) propose another solution to the over-determination problem by considering investment allocation. In their model, the ratio between investment goods allocated to different sectors is endogenously determined.

Kim and Lavoie (2017) and Araujo and Drumond (2020) provide new insights to the over-determination problem. However, their solutions are not entirely satisfactory for several reasons. In Kim and Lavoie's (2017) model, the target rates of return adjust to the short-run rates of profit in equilibrium (adjustment mechanism 3). The target rate of return in a sector is the rate of profit obtained when capital is normally utilised, which means that it is the normal rate of profit. Usually, the normal rate of profit is not the same as the short-run profit rate in equilibrium. However, the normal rate of profit should be the centre of gravity of the actual rate of profit, not the other way around. With regards to Araujo and Drumond (2020), if the allocation of investment goods into different sectors is fixed, then capital cannot move freely among sectors, and it is odd that the rates of profit will be equalised. Indeed, capital does not move among sectors only when the rates of profit have been equalised. Therefore, the fixed ratio of investment allocation is not determined *a priori* but is a result of equalised rates of profit. What really matters for Araujo and Drumond's (2020) solution is that the relative price of two sectors is endogenous.

In the following section, we also seek to solve the over-determination problem by introducing another endogenous variable. Before we propose the solution, we first clarify some issues.

4. Mark-up rates and the equalization on the profit rate

In another paper, Dutt (1987) uses a similar model to argue that the existence of monopoly power and classical competition with uniform rate of profit are not inconsistent. Glick and Campbell (1995) and Duménil and Lévy (1995) have criticised Dutt's model. Glick and Campbell (1995) argue that Dutt (1987) failed to provide a meaningful dynamic model by which monopoly power is compatible with equalised profit rate. Duménil and Lévy (1995) contend that Dutt's (1987) investment function is deficient, and that the variables treated as given in a short period must be treated endogenously. Taking these debates into account, the over-determination problem concerns not only the question of whether the equalisation of profit rate is compatible with sectoral investment functions but also whether it is compatible with the case when firms have monopoly power. In Dutt's model, mark-up pricing theory reflects the monopoly power, and therefore another related question is whether classical competition is compatible with mark-up pricing.

First, there is no conflict between classical free competition characterised by uniform rate of profit and mark-up pricing. Typically, the mark-up pricing used in neo-Kaleckian theory reflects that firms have monopoly power. Indeed, Kalacki's microanalysis, which makes up the core of mark-up pricing theory (Lee, 2006), focuses mainly on oligopolies. However, mark-up pricing is not limited to the theory of monopoly or oligopoly, as mark-up pricing is not a price theory, but rather a pricing theory, which discusses how price decisions are actually taken (Lavoie, 2014). Under classical free competition, firms also set prices using their market power (Salvadori and Signorino, 2013). Free competition must not be confused with the neo-classical perfect competition: under perfect competition, atomic agents treat prices parametrically, and no single firm can change the price; in contrast, under free competition, firms are price setters rather than price takers. Therefore, mark-up pricing theory is compatible with the classical theory of free competition.

Second, the equalised rate of profit in the classical theory is not a final state but rather a process. In the classical theory, the equalised rate of profit is a tendency: if rates of profit are not equalised, such as if the rate of profit in one industry is higher than that in others (i.e., firms in that industry can earn extra profit), then this one industry will attract capital. This increased investment will cause a rise in supply, which further leads to a fall in price and the rate of profit. Conversely, an industry with a lower profit rate will experience an outflow of capital and a decrease in investment, which will further lead to a rise in price and the profit rate. Capital will continue to enter industries with higher profit rates and exit industries with lower profit rates, provided there is no entry/exit barriers to prevent such movements. In the long run, movements of capital will equalise the rate of profit, and the prices of different commodities will tend to their natural prices or prices of production. The process of profit equalisation may never occur, as market prices may never equal the prices of production, but this does not mean that the classical free competition concept is useless. The prices of production are centres of gravity that market prices continue to approach. The equalisation of profit is also a tendency.

For example, in a widely cited paragraph, Ricardo wrote:

> Whilst every man is free to employ his capital where he pleases, he will naturally seek for it that employment which is most advantageous; he will naturally be dissatisfied with a profit of 10 per cent., if by removing his capital he can obtain a profit of 15 per cent. This restless desire on the part of all the employers of stock, to quit a less profitable for a more advantageous business, has a strong tendency to equalize the rate of profits of all, or to fix them in such proportions, as may in the estimation of the parties, compensate for any advantage which one may have, or may appear to have over the other (Ricardo, 2004, pp.88-89).

Third, under free competition, the mark-up rates of different sectors cannot be given arbitrarily and independently. Under free competition, if firms set up higher prices (a higher mark-up rate) such that they earn extra profit, then the extra profit will draw other firms to invest in this industry. Consequently, supply will increase and competition in this industry will lead firms to reduce their prices; that is, they will reduce their mark-up rate. The opposite happens in other industries: capital outflows will decrease supply and firms will set higher prices; that is, raise their mark-up rate. Therefore, under free competition, firms cannot set prices and do not consider other firms' pricing behaviour.

Finally, the classical theory does not stick to the uniform rate of profit. If barriers prevent the movement of capital, then the rates of profit can be persistently different. Smith pointed out many cases in which monopolists obtain higher profit than they would under free competition (Salvadori and Signorino, 2014). Marx (1991, p. 312) also wrote that if there exist higher risks or slow capital turn over caused by distant markets, then capitalists will take these situations into their calculations and increase their prices as compensation. In other words, the classical theory does not neglect the cases of monopolies and those with non-equalised profit rates. If the monopoly power or the factors leading to non-equalised profit rates are stable, then the differences between the rates of profit also remain stable, and so the differences between the mark-up rates in different sectors will differ.

5. Proposed alternative solution to the over-determination problem

In section 4, we argue that firms cannot set their prices independently when free competition applies; that is, there is a relationship between the mark-up rates of different sectors. This idea is related to our proposed solution to the over-determination problem. In this section, we introduce another variable but without changing the number of core equations. Besides System (1), we assume the following equation holds:

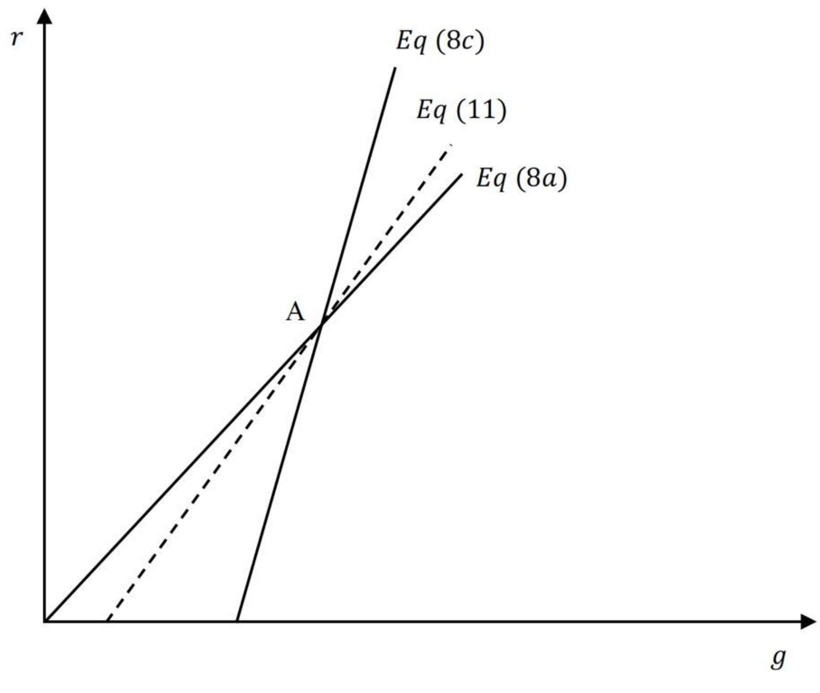
$$\tau_1 = \delta\tau_2 \tag{10}$$

where δ is the relative mark-up ratio. By replacing τ_1 with $\delta\tau_2$ in System (1), the system has 11 equations and 11 unknown variables and is perfectly determined. As for System (8), Equation (8b) becomes

$$g_1 = \alpha_1 + (\beta_1 + \gamma_1 \frac{1+\tau_2}{\delta\tau_2}\frac{l_2}{l_1}v_1)r_1 \tag{11}$$

Equations (8a) and (8c) still hold, and these two straight lines in the (g, r) quadrant have one and only one intersection, which determines $g^* = g_1^* = g_2^*$ and $r^* = r_1^* = r_2^*$. Once (g^*, r^*) is determined, (point A in Figure 2), Equation (11) determines δ^* such that the straight line (Equation (11)) intersects with point A.

Figure 2. Relative mark-up ratio and solution to the over-determination problem



The solutions to System (1) are as follows:

$$r_1^* = r_2^* = \frac{\alpha_2\tau_2}{(s_c - \beta_2)\tau_2 + \gamma_2(1+\tau_2)v_2} \tag{12a}$$

$$g_1^* = g_2^* = \frac{s_c\alpha_2\tau_2}{(s_c - \beta_2)\tau_2 + \gamma_2(1+\tau_2)v_2} \tag{12b}$$

$$u_1^* = \frac{\alpha_2\tau_2}{y_1\{[\alpha_2(s_c - \beta_1) - \alpha_1(s_c - \beta_2)]\tau_2 - \alpha_1\gamma_2(1+\tau_2)v_2\}} \tag{12c}$$

$$u_2^* = \frac{\alpha_2(1+\tau_2)v_2}{(s_c - \beta_2)\tau_2 + \gamma_2(1+\tau_2)v_2} \tag{12d}$$

$$\delta^* = \frac{[y_1(1+\tau_2)v_1]\{[\alpha_2(s_c - \beta_1) - \alpha_1(s_c - \beta_2)]\tau_2 - \alpha_1\gamma_2(1+\tau_2)v_2\}}{(s_c - \beta_2)\tau_2 + \gamma_2(1+\tau_2)v_2} \tag{12e}$$

$$p_1^* = (1 + \frac{[y_1(1+\tau_2)v_1]\{[\alpha_2(s_c - \beta_1) - \alpha_1(s_c - \beta_2)]\tau_2 - \alpha_1\gamma_2(1+\tau_2)v_2\}}{(s_c - \beta_2)\tau_2 + \gamma_2(1+\tau_2)v_2})Wl_1 \tag{12f}$$

$$p_2^* = (1+\tau_2)Wl_2 \tag{12g}$$

$$w^* = \frac{(s_c - \beta_2)\tau_2 + \gamma_2(1+\tau_2)v_2}{\gamma_1(1+\tau_2)v_1\alpha_2(s_c - \beta_1)l_1 + [1 - \gamma_1(1+\tau_2)v_1\alpha_1][(s_c - \beta_2)\tau_2 + \gamma_2(1+\tau_2)v_2 l_1]} \tag{12h}$$

$$x^* = \frac{s_c\gamma_1(1+\tau_2)v_1\{[\alpha_2(s_c - \beta_1) - \alpha_1(s_c - \beta_2)]\tau_2 - \alpha_1\gamma_2(1+\tau_2)v_2\}}{[(s_c - \beta_2)\tau_2 + \gamma_2(1+\tau_2)v_2](1 + \tau_2 - s_c\tau_2)} \tag{12i}$$

Up to now, we show that over-determination does not exist in the long run, and that the rates of profit can be equalised in the steady state in a two-sector neo-Kaleckian model. However, how does the model adjust from the short-run equilibrium to the long-run steady state?

6. Adjustment mechanism from the short-run to the long-run equilibrium

Neo-Kaleckian theory usually distinguishes the short-run analysis from the long-run analysis. The usual assumption is that capital in different sectors ($K_1$and K_2) are taken as given in the short run and that they grow in the long run due to capital accumulation. In this section, we also adopt this assumption. Additionally, we assume that the mark-up rates in different sectors (τ_1 and τ_2) are given in the short run (hence, δ is also given in the short run), and they can change in the long run. We take the investment function as the linear form as Equations (4) and (5). In the short run, Equations (1a) to (1g) and Equations (4) and (5) hold. Together with $X_i = u_i \frac{K_i}{v_i}$, we have 11 equations and 11 variables (p_i, w, g_i, r_i, u_i, and X_i), and the model is perfectly determined in the short run. To focus on the adjustment mechanism of g_i and r_i from the short run to the long run, we show the solutions of these 4 variables only for the short-run equilibrium, as is in System (13).

$$r_1^* = \frac{\alpha_1 + \alpha_2 k}{\left[\frac{C}{C-1} - (\beta_2 + BC)\frac{1}{C-1} - (\beta_1 + AC\frac{1}{\delta})\right]} \tag{13a}$$

$$r_2^* = \frac{\alpha_1 + \alpha_2 k}{(C-1)k\left[\frac{C}{C-1} - (\beta_2 + BC)\frac{1}{C-1} - (\beta_1 + AC\frac{1}{\delta})\right]} \tag{13b}$$

$$g_1^* = \alpha_1 + (\beta_1 + AC\frac{1}{\delta})\frac{\alpha_1 + \alpha_2 k}{\left[\frac{C}{C-1} - (\beta_2 + BC)\frac{1}{C-1} - (\beta_1 + AC\frac{1}{\delta})\right]} \tag{13c}$$

$$g_2^* = \alpha_2 + (\beta_2 + BC\frac{1}{\delta})\frac{\alpha_1 + \alpha_2 k}{(C-1)k\left[\frac{C}{C-1} - (\beta_2 + BC)\frac{1}{C-1} - (\beta_1 + AC\frac{1}{\delta})\right]} \tag{13d}$$

where $k = \frac{K_2}{K_1}$, $A = \gamma_1 \frac{l_2}{l_1} v_1$, $B = \gamma_2 v_2$, and $C = \frac{1+\tau_2}{\tau_2} = 1 + \frac{1}{\tau_2}$.

In the long-run, capital stock and δ change. Based on the reasoning in the former section, we assume the following adjustment mechanism for δ:

$$\dot{\delta} = \lambda(r_2 - r_1) \tag{14}$$

where λ is a positive coefficient. The economic meaning of Equation (14) is, if r_2 is higher than r_1, then competition will cause an inflow of capital in Sector 2 and an outflow of capital in Sector 1. Consequently, τ_2 will be lower and τ_1 will be higher. Therefore, δ will be higher.

Since $k = \frac{K_2}{K_1}$, we have:

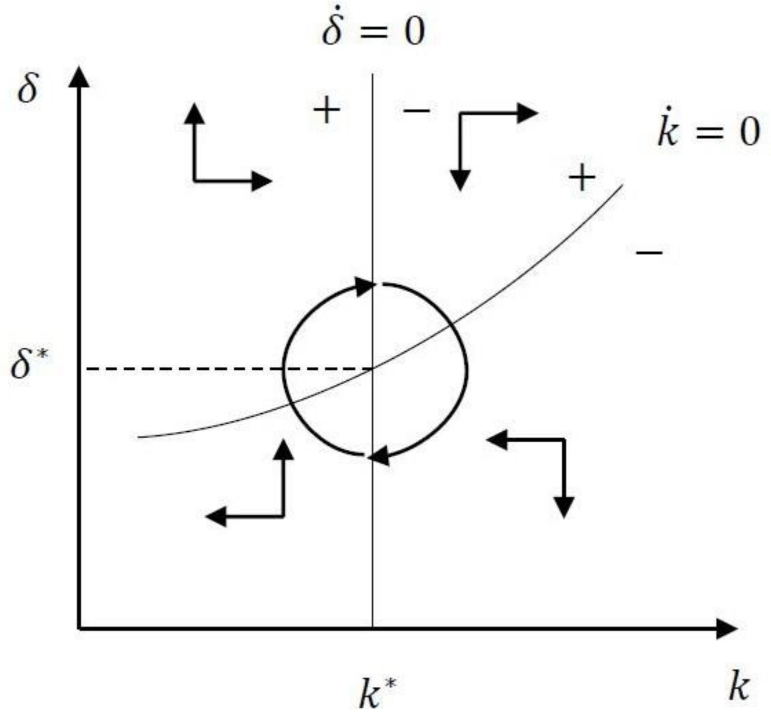
$$\dot{k} = g_2 - g_1 \tag{15}$$

In the long-run equilibrium, $\dot{k} = 0$, $\dot{\delta} = 0$. In Section 5, we show that such an equilibrium exists, but is it stable? In the following, we use a phase diagram to investigate the stability of the long-run equilibrium.

From Equations (13a) and (13b), we find that for $\dot{\delta} = 0, k = \tau_2$. Therefore, for $\dot{\delta} = 0$, $\delta = f(k)$ is a vertical line in the (δ, k) phase. Additionally, we observe that $\frac{d\dot{\delta}}{dk} < 0$. From Equations (13c) and (13d), we find that for $\dot{k} = 0$, δ is a function of k: $\delta = h(k)$. It can be shown that $\frac{dh(k)}{dk} > 0$. Furthermore, we can show that $\frac{d\dot{k}}{d\delta} > 0$.

As Figure 3 illustrates, the dynamics of δ and k form a circle orbiting the equilibrium point. It appears that the long-run equilibrium is not stable and the equalisation of the rates of profit and growth can never occur because the short run dynamics do not converge to the long-run equilibrium. Hence, it also appears that the short-run price does not converge to the long-run equilibrium price. However, as we argued in Section 4, classical competition does not mean that the market price converges to the price of production. The latter is a centre of gravity which the former continues to approach. Asking for the market price to converge to the price of production is asking too much. The equalisation of profit rates is also not a final state but a process; it may not be achieved but is a tendency for the actual rates of profit. Therefore, the circle in Figure 3 illustrates exactly the importance of the equilibrium.

Figure 3. Adjustment mechanism from the short-run to the long-run equilibrium



7. Concluding remarks

In this paper, we aim to solve the over-determination problem in the two-sector neo-Kaleckian model. What really matters for the over-determination problem is not the incompatibility of sectoral investment functions and the equalised profit rate, but rather the arbitrarily and independently given mark-up rates. Once we introduce the relative mark-up ratio between different sectors, the model is perfectly determined and logically consistent. We also discuss the adjustment mechanism from the short-run to the long-run steady state. The dynamics of the mark-up ratio and relative capital size form a circle from the short to the long run. However, this circle does not contradict the classical free competition concept. It illustrates exactly the importance of the long run equilibrium.

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